Homework 4

1. Consider the Cauchy problem

$$\begin{cases} u_t = bu_{xx} & \text{for } x \in \mathbb{R},\ t \in [0, \infty), \\ u(0,x) = u_0(x) & \text{for } x \in \mathbb{R}, \end{cases}$$

   where b is a positive constant. We have proved in class

$$\int_{-\infty}^{\infty} |u(t,x)|^2 \, dx + 2b \int_0^t \int_{-\infty}^{\infty} |u_x(\tau,x)|^2 \, dx \, d\tau = \int_{-\infty}^{\infty} |u(0,x)|^2 \, dx,$$

   by using the energy method. Prove the above equation using the Fourier transformation.

2. Textbook, 6.3.1 (only do the part for (6.3.4)).

3. Textbook, 6.3.5, 6.3.9, 6.3.14.