Homework 5 Due 12/10/2015

1. (20 points) Consider the 2D convection-diffusion equation

$$\begin{cases} u_t + \mathbf{a}\cdot\nabla u = b\Delta u + 1 & \text{for } (x,y)\in(0,1)^2,\ t\in(0,T) \\ u(0,x,y) = 0 & \text{for } (x,y)\in(0,1)^2 \\ u(t,x,y) = 0 & \text{for } (x,y) \text{ lying on the boundary of the square } (0,1)^2 \end{cases}$$

where

$$\mathbf{a}\cdot\nabla u = a_1 u_x + a_2 u_y, \qquad \Delta u = u_{xx} + u_{yy}$$

Here we consider a convection dominant problem with

$$\mathbf{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad b = 0.001$$

(a) Use the forward-time central space scheme to compute an approximation to $u(1,x,y)$ with $h = 0.1$, 0.05, 0.01 and $k = h^2$. Draw the graph of your numerical solution using the `surf` command in Matlab, and report your observations.

(b) Design an "upwinding" scheme for this problem and repeat step (a).

(c) Repeat (a)-(b) for $b = 10^{-6}$.

(Not counted towards your score) Think about the following question: what if vector $\mathbf{a}$ is not $(1,0)^t$, or what if $\mathbf{a} = \mathbf{a}(x,y)$ is not a constant vector?