#2 Recall that

$$\widehat{\delta_0 v_m} = \frac{i\sin\theta}{h}\,\widehat{v_m}$$

By discrete Fourier ~~than~~ transform one gets

$$\frac{g^2-1}{2k} + a\,\frac{i\sin\theta}{h}\cdot\frac{g^2+4g+1}{6} = 0$$

$$\Rightarrow \quad g^2-1 + i\,\frac{a\lambda\sin\theta}{3}\,(g^2+4g+1) = 0$$

Setting $b = \frac{a\lambda\sin\theta}{3}$, one gets

$$(1+ib)\,g^2 + 4ib\,g + (-1+ib) = 0$$

$$\Rightarrow \quad g_{1,2} = \frac{-2ib \pm \sqrt{1-3b^2}}{1+ib}$$

Case 1: If $|a\lambda| < \sqrt{3}$, we have $|b| < \frac{1}{\sqrt{3}} \Rightarrow 1-3b^2 > 0$

$\Rightarrow \sqrt{1-3b^2}$ is real

Then, $g_1 \neq g_2$, moreover,

$$|g_1|^2 = |g_2|^2 = \frac{4b^2 + (1-3b^2)}{1+b^2} = 1 \Rightarrow \text{stable}$$

Case 2: If $|a\lambda| = \sqrt{3}$, ~~when $\theta \neq \pm\pi$, we still get $|b| < \frac{1}{\sqrt{3}}$ and the analysis is the~~ Let $\theta = \pi$ or $-\pi$, then $|b| = \frac{1}{\sqrt{3}}$ and hence $g_1 = g_2 = \frac{-2ib}{1+ib}$. Moreover, $|g_1|^2 = |g_2|^2 = \frac{4b^2}{1+b^2} = \frac{4/3}{1+1/3} = 1$

In other words, we have a repeated root with modulus 1

$\Longrightarrow$ unstable

Case 3 If $|a\lambda| > \sqrt{3}$, set $\theta = \pi$ or $-\pi$.

then $|b| > \frac{1}{\sqrt{3}}$ and hence $1-3b^2 < 0$.

Consequently, $\sqrt{1-3b^2} = i\sqrt{3b^2-1}$

where $\sqrt{3b^2-1}$ is real.

In this case, $g_1 \neq g_2$ and

$$\max\left(|g_1|^2, |g_2|^2\right) = \max\left(\frac{\left(-2b+\sqrt{3b^2-1}\right)^2}{1+b^2}, \frac{\left(-2b-\sqrt{3b^2-1}\right)^2}{1+b^2}\right)$$

Without loss of generality, consider the case $b>0$, then

$$\max\left(|g_1|^2, |g_2|^2\right) = \frac{\left(-2b-\sqrt{3b^2-1}\right)^2}{1+b^2}$$

$$= \frac{7b^2-1+4b\sqrt{3b^2-1}}{1+b^2}$$

$$= \frac{(1+b^2)+\left[6b^2-2+4b\sqrt{3b^2-1}\right]}{1+b^2}$$

$$= 1 + \frac{2(3b^2-1)+4b\sqrt{3b^2-1}}{1+b^2}$$

$$> 1 \quad (\text{Since } b>0 \text{ and } 3b^2-1>0)$$

$\Longrightarrow$ unstable

(For $b<0$, the analysis is similar.)