Homework 3 Due 10/24/2013

1. (6 points) Textbook 5.2.2

2. (6 points) Show that the Du Fort-Frankel scheme for $u_t = bu_{xx}$, $b > 0$, is consistent only when $k/h \to 0$ as k and h goes to 0.

3. (8 points) Consider the following equation

$$\begin{cases} u_t = \frac{1}{\pi^2} u_{xx} & \text{for } 0 \le x \le 1 \text{ and } t \ge 0 \\ u(0, x) = \sin \pi x & \text{for } 0 \le x \le 1 \\ u(t, 0) = u(t, 1) = 0 & \text{for } t \ge 0 \end{cases}$$

The exact solution is $u = e^{-t} \sin \pi x$.

For $h = 0.1, 0.05, 0.025, 0.0125$, use the following designated schemes and refinement paths to compute the numerical solution at $t = 1$. Compute the error in 2-norm at $t = 1$ and plot the error-vs-h graphs.

   (a) Use Forward time central space with $k = h$;

   (b) Use Forward time central space with $k = h^2$;

   (c) Use Crank-Nicolson with $k = h$.