Project: Burgers equation

Consider the inviscid Burger's equation

$$u_t + uu_x = 0$$

or the viscid Burger's equation

$$u_t + uu_x = \varepsilon u_{xx}$$

where ε is a positive number.

Consider the following questions. You team is expected to submit a project report containing both theoretical survey and numerical results, and to make a classroom presentation.

1. Read from various sources about the property of Burger's equations, the characteristic lines, etc.

   Suggested reading: Partial Differential Equations by L.C. Evans, Section 3.4
   Suggested reading: Numerical Methods for Conservation Laws by R.J. LeVeque, Section 3.2

2. Try to apply finite difference schemes learned in class to the Burger's equation. Can you find a working method?

3. Burger's equation is known to be difficult to solve. One solution is to use the Godunov's method.

   Suggested reading: Numerical Methods for Conservation Laws by R.J. LeVeque, Section 13.2, 13.5

   Take the inviscid Burger's equation as an example. It can be rewritten into a conservation form

   $$u_t + f_x = 0, \quad \text{where } f = \frac{u^2}{2}$$

   The scheme given in LeVeque's book has the form:

   $$\frac{U_m^{n+1} - U_m^n}{k} + \frac{F_{m+\frac{1}{2}}^n - F_{m-\frac{1}{2}}^n}{h} = 0$$

   where the numerical flux is defined by

   $$F_{m+\frac{1}{2}}^n = \frac{(\bar{U}_{m+\frac{1}{2}}^n)^2}{2}$$

   in which

   (a) If $U_m^n \geq U_{m+1}^n$

   $$\bar{U}_{m+\frac{1}{2}}^n = \begin{cases} U_m^n, & \text{if } (U_m^n + U_{m+1}^n)/2 > 0 \\ U_{m+1}^n, & \text{if } (U_m^n + U_{m+1}^n)/2 \leq 0 \end{cases}$$

   (b) If $U_m^n < U_{m+1}^n$

   $$\bar{U}_{m+\frac{1}{2}}^n = \begin{cases} U_m^n, & \text{if } U_m^n > 0 \\ U_{m+1}^n, & \text{if } U_{m+1}^n < 0 \\ 0, & \text{if } U_m^n \leq 0 \leq U_{m+1}^n \end{cases}$$