Programming assignment 4

1. Solve the Poisson's equation in $\Omega = (0,1) \times (0,1)$:

$$\begin{cases} -\Delta u = 2\pi^2 \sin(\pi x)\sin(\pi y) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

by the standard five-point finite difference scheme on a uniform $M \times M$ grid. The step size is $h = 1/M$. The exact solution for this problem is $u = \sin(\pi x)\sin(\pi y)$.

   (a) Compute the maximum norm of error $\|u - v\|_\infty$ on grid points, where v is the finite difference solution. Report the error for 16×16, 32×32, 64×64 and 128×128 grids. Does your result agree with the error estimate

$$\|u - v\|_\infty = O(h^2).$$

   (b) Plot the numerical solution on a 20×20 grid.

Note:

- You may use the Matlab build-in function "delsq" to generate the stiffness matrix. However, you will need to read the help file and make sure you use it correctly.
- To solve a linear system $Ax = f$, where x, and f are n-dim column vectors, you can use the Matlab command "x = A\ f".
- A useful Matlab command is "reshape", which returns a matrix whose elements are taken column-wise from a given vector or matrix. For example, let column vector $x = [1,2,3,4]^t$. "y=reshape(x,2,2)" will return a 2×2 matrix $y = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Then you can use command "surf(y)" to plot the matrix as a surface.