

Programming assignment 3

1. Consider the following problem:

$$\begin{cases} u_t + u_x = \varepsilon u_{xx} & \text{for } x \in [0, 1], t \geq 0, \\ u(0, x) = 1 - x & \text{for } x \in [0, 1], \\ u(t, 0) = 1, \quad u(t, 1) = 0. \end{cases}$$

- (a) Set $\varepsilon = 1$, solve the problem using the forward-time central-space scheme with $h = 1/20$, $\mu = k/h^2 = 0.5$. Plot the solution at $T = 0.5$. Then solve the problem with the same settings except for $\mu = k/h^2 = 0.6$, plot the solution at around $T = 0.5$. Explain the difference you observe between these two solutions.
- (b) Set $\varepsilon = 0.01$. Solve the problem using the forward-time central-space scheme with $h = 1/20$, $\mu = k/h^2 = 0.2$. Plot the solution at $T = 2.0$. Then solve the problem with the same settings except for $h = 1/60$, plot the solution at $T = 2.0$. Explain the difference you observe between these two solutions.
- (c) Repeat part (a) using the forward-time backward-space (upwinding) scheme, and $\mu = 0.4, 0.5$. Explain the difference between solutions.
- (d) Repeat part (b) using the forward-time backward-space (upwinding) scheme. What do you observe?