Math 4553, Homework 2, Due on 2/17/2014

1. Consider the problem

$$\begin{aligned} \min \quad & f = -x - 0.5y \\ \text{subject to} \quad & 2x + y \le 8 \\ & x - 4y \le 1 \\ & x \ge 0,\ y \ge 0 \end{aligned}$$

   (a) (4 points) Solve the problem using the simplex method. Does the problem have multiple solutions?

   (b) (4 points) Solve the problem using graphical optimization. In the graph, denote the vertices corresponding to each step in the simplex method, and trace the path of the simplex method.

2. (4 points) Rewrite the following linear programming problem into the standard form

$$\begin{aligned} \min \quad & f = 2x_1 + x_2 - 3x_3 \\ \text{subject to} \quad & x_1 - 2x_2 + x_3 = 10 \\ & x_1 + x_2 \ge 4 \\ & 2 \le x_1 \le 8 \\ & x_3 \ge 0 \end{aligned}$$

3. (4 points) Use Phase I procedure to demonstrate that

$$\begin{aligned} \min \quad & f = -3x_1 + x_2 \\ \text{subject to} \quad & -x_1 - x_2 \ge -2 \\ & 2x_1 + 2x_2 \ge 10 \\ & x_1 \ge 0,\ x_2 \ge 0 \end{aligned}$$

   is infeasible.

4. (4 points) Use the simplex method to demonstrate that

$$\begin{aligned} \min \quad & f = -x_1 + x_2 \\ \text{subject to} \quad & 2x_1 - x_2 \ge 1 \\ & x_1 + 2x_2 \ge 2 \\ & x_1 \ge 0,\ x_2 \ge 0 \end{aligned}$$

   is unbounded. Moreover, find a feasible point (x_1, x_2) such that $f(x_1, x_2) = -650$.