Math 4553, Homework 1, Due on 2/3/2014

1. (4 points) A company can produce three different types of concrete blocks, identified as A, B, and C. The production process is constrained by facilities available for mixing, vibration, and inspection/drying. Using the data given in the following table, formulate the production problem in order to maximize the profit.

	A	B	C	available
Mixing (hours/batch)	1	3	9	900
Vibration (hours/batch)	2	3	6	1,200
Inspection/drying (hours/batch)	0.7	0.8	1	400
Profit ($/batch)	7	17	30	

2. (4 points) A company requires open-top rectangular containers to transport material. Using the following data, formulate an optimum design problem to determine the container dimensions for minimum cost.

Construction costs	Sides = $\$65/m^2$, Bottom = $\$120/m^2$
Can be used for	10 years
Yearly maintenance cost	$\$12/m^2$ of the outside surface area
Minimum volume of the container	$1200\,m^3$

3. (6 points) Use graphical methods to solve the following optimization problem:

$$\text{Maximize } f(x,\, y) = -6x + 9y$$
$$\text{Subject to } \begin{pmatrix} x - y \geq 2 \\ 3x + y \geq 1 \\ 2x - 3y \geq 3 \end{pmatrix}$$

(Notice that x, y are not necessarily positive in this problem.)

4. (6 points) Consider the linear system

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -4 & 13 \\ -4 & 1 & 23 \\ 3 & -6 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

(a) Perform the Jordan exchange of y_3 and x_2, and derive the new linear system

$$\begin{pmatrix} y_1 \\ y_2 \\ x_2 \\ y_4 \\ y_5 \\ y_6 \end{pmatrix} = B \begin{pmatrix} x_1 \\ y_3 \\ x_3 \end{pmatrix}$$

Compute B.

(b) After this, perform the Jordan exchange of y_4 and x_1, and derive the new linear system

$$\begin{pmatrix} y_1 \\ y_2 \\ x_2 \\ x_1 \\ y_5 \\ y_6 \end{pmatrix} = C \begin{pmatrix} y_4 \\ y_3 \\ x_3 \end{pmatrix}$$

Compute C.

(Please write the details in order to get full credit.)