Math 4553, Exam II, Apr. 9, 2012 Name: ___________________

1. (12 points) Write the dual problem for the following linear programming problem

$$\begin{array}{ll} \text{min} & f = 3x_1 + 4x_2 + 5x_3 + 6x_4 \\ \text{subject to} & x_1 - x_2 + 4x_3 + 6x_4 \le 0 \\ & x_1 + 2x_2 + 2x_3 + x_4 = 9 \\ & 5x_1 - 6x_2 - 7x_3 + 8x_4 \ge 3 \\ & x_2 \ge 0,\ x_3 \ge 0 \\ & x_1,\ x_4 \text{ are free variables} \end{array}$$

2. Consider the standard form for the quadratic programming problem

$$\begin{array}{ll} \text{min} & f = \frac{1}{2}\mathbf{x}^t Q \mathbf{x} + \mathbf{p}^t \mathbf{x} \\ \text{subject to} & A\mathbf{x} \ge \mathbf{b} \\ & \mathbf{x} \ge \mathbf{0} \end{array}$$

(a) (8 points) Denote $\mathbf{x}$ and $\mathbf{u}$ to be the primal and the dual variables, respectively. Write down the KKT conditions for the quadratic problem.

(b) (10 points) Given $Q = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, $p = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \end{bmatrix}$, and $b = \begin{bmatrix} 2 \end{bmatrix}$. Clearly the primal and dual variables have dimension $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} u_1 \end{bmatrix}$. It is given to you that minimum value of f occurs at $x_1 = ?$, $x_2 = 2$ and $u_1 = 1$. Use the KKT conditions to find the value of x_1 and $\min f$. (Remark: You will NOT get credit if you use Lemke's method to solve this problem.)

3. (20 points) Determine whether the following quadratic programming problem is convex or not,

$$\begin{array}{ll} \text{min} & f = x_1^2 + x_2^2 + x_1x_2 - x_1 - x_2 \\ \text{subject to} & x_1 + x_2 - 2 \ge 0 \\ & 2x_1 - 1 \ge 0 \\ & x_1,\ x_2 \ge 0 \end{array}$$

Then, use the Lemke's method to solve the quadratic problem