1. **Math 4553, Exam I,** Feb. 24, 2012, Solution

$$\begin{array}{rl} \min & f = 260A_1 + 410A_2 + 350A_3 + 380B_1 + 340B_2 + 390B_3 + 220C_1 + 370C_2 + 440C_3 \\ \text{subject to} & A_1 + A_2 + A_3 = 70 \\ & B_1 + B_2 + B_3 = 90 \\ & C_1 + C_2 + C_3 = 50 \\ & A_1 + B_1 + C_1 \le 35 \\ & A_2 + B_2 + C_2 \le 120 \\ & A_3 + B_3 + C_3 \le 80 \\ & A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3 \ge 0 \end{array}$$

2. The graph is omitted. The minimum value of f is $\frac{34}{3}$ which is taken at $x = \frac{5}{3}$ and $y = \frac{4}{3}$.

3. The problem is not in standard form. We will use scheme II. The initial tableau is

```
              x1          x2          x3           1
          ------------------------------------------
 y1 = |     1.0000     -1.0000      3.0000     -3.0000
 y2 = |     4.0000      1.0000      0.0000     -1.0000
          ------------------------------------------
 f  = |     1.0000      2.0000      3.0000      0.0000
```

Note that x_2 is a free variable and y_1 correspond to an equation type constraint. Use Jordan exchange to swap x_2 and y_1, we have

```
              x1          y1          x3           1
          ------------------------------------------
 x2 = |     1.0000     -1.0000      3.0000     -3.0000
 y2 = |     5.0000     -1.0000      3.0000     -4.0000
          ------------------------------------------
 f  = |     3.0000     -2.0000      9.0000     -6.0000
```

Now delete the y_1 column and move the x_2 row to the bottom. This gives

```
              x1          x3           1
          --------------------------------
 y2 = |     5.0000      3.0000     -4.0000
          --------------------------------
 f  = |     3.0000      9.0000     -6.0000
 x2 = |     1.0000      3.0000     -3.0000
```

Notice this tableau is not feasible. Therefore we must start the phase I process, by adding artificial variable x_0 and a new objextive function f_0.

	x1	x3	x0	1
y2 =	5.0000	3.0000	1.0000	-4.0000
f =	3.0000	9.0000	0.0000	-6.0000
x2 =	1.0000	3.0000	0.0000	-3.0000
f0 =	0.0000	0.0000	1.0000	0.0000

Perform a special pivot with x_0 as the pivot column and y_2 as the pivot row:

	x1	x3	y2	1
x0 =	-5.0000	-3.0000	1.0000	4.0000
f =	3.0000	9.0000	0.0000	-6.0000
x2 =	1.0000	3.0000	0.0000	-3.0000
f0 =	-5.0000	-3.0000	1.0000	4.0000

Next, try to minimize function f_0 by using Jordan exchanges. Obviously, we should pick x_1 as the pivot column and x_0 as the pivot row:

	x0	x3	y2	1
x1 =	-0.2000	-0.6000	0.2000	0.8000
f =	-0.6000	7.2000	0.6000	-3.6000
x2 =	-0.2000	2.4000	0.2000	-2.2000
f0 =	1.0000	0.0000	0.0000	0.0000

The tableau is optimal in terms of f_0 and the minimum value of f_0 is 0. This means we have found a feasible solution. Delete the f_0 row and the x_0 column and this gives the starting tableau for Phase II:

	x3	y2	1
x1 =	-0.6000	0.2000	0.8000
f =	7.2000	0.6000	-3.6000
x2 =	2.4000	0.2000	-2.2000

This tableau is indeed optimal. Hence we have the optimal solution, which is

$$x_1 = 0.8, \quad x_2 = -2.2, \qquad x_3 = 0$$

and

$$\min f = -3.6$$