Math 4553, Homework 2, Due on 2/17/2012

1. Consider the problem

$$\begin{aligned}
\min \quad & f = -x - 0.5y \\
\text{subject to} \quad & 2x + 3y \leq 12 \\
& 2x + y \leq 8 \\
& x - 4y \leq 1 \\
& x \geq 0, \ y \geq 0
\end{aligned}$$

   (a) (4 points) Solve the problem using the simplex method. Does the problem have multiple solutions?

   (b) (4 points) Solve the problem using graphical optimization. In the graph, denote the vertices corresponding to each step in the simplex method, and trace the path of the simplex method.

2. (6 points) Rewrite the following linear programming problem into the standard form

$$\begin{aligned}
\min \quad & f = 2x_1 + x_2 - 3x_3 \\
\text{subject to} \quad & x_1 - 2x_2 + x_3 = 10 \\
& x_1 + x_2 \geq 4 \\
& 2 \leq x_1 \leq 8 \\
& x_3 \geq 0
\end{aligned}$$

3. Consider the following two LP problems:

$$\begin{aligned}
\min \quad & f = -6x_2 \\
\text{subject to} \quad & -3x_1 - 4x_2 \geq -7 \\
& 2x_1 - 5x_2 \geq -1 \\
& x_1 \geq 0, \ x_2 \geq 0
\end{aligned}$$

   and

$$\begin{aligned}
\max \quad & g = -7y_1 - y_2 \\
\text{subject to} \quad & -3y_1 + 2y_2 \leq 0 \\
& -4y_1 - 5y_2 \leq -6 \\
& y_1 \geq 0, \ y_2 \geq 0
\end{aligned}$$

   (a) (4 points) Compare these two problems. State all relations you have observed between these two problems.

   (b) (2 points) Use graphical optimization to solve these two problems, separately. Is there any relation between the solutions?