Math 4553, Exam II, Apr. 18, 2011 Name: ___________________

Formula:

- If $\bar{\mathbf{x}}$ is a local solution to the optimization problem *min* $f(\mathbf{x})$, *subject to* $x \in S$, where S is a convex set, then $\bar{\mathbf{x}}$ satisfies $\bar{\mathbf{x}} \in S$ and $\nabla f(\bar{\mathbf{x}}) \cdot (\mathbf{x} - \bar{\mathbf{x}}) \geq 0$ for all $\mathbf{x} \in S$.
- (KKT conditions for quadratic programming) If $\bar{\mathbf{x}}$ is a local solution to the quadratic programming problem, then there exists $\bar{\mathbf{u}}$ such that $0 \leq \bar{\mathbf{x}} \perp Q\bar{\mathbf{x}} - A'\bar{\mathbf{u}} + \mathbf{p} \geq 0$ and $0 \leq \bar{\mathbf{u}} \perp A\bar{\mathbf{x}} - \mathbf{b} \geq 0$.
- Given a vector $\mathbf{v}$ and a matrix A, we have

$$proj_{null(A)}\mathbf{v} = \left(I - A^T(AA^T)^{-1}A\right)\mathbf{v}$$

- (PAS algorithm) Given the k-th interior point $\mathbf{x}^k$, compute the affine transformation $\mathbf{x}^k = T^k\mathbf{y}^k$, where $\mathbf{y}^k = (1, \ldots, 1)^t$. Use this affine transformation to rewrite the linear programming problem into *minimize* $f = \mathbf{p}^k\mathbf{y}$, *subject to* $A^k\mathbf{y} = \mathbf{b}$, $\mathbf{y} \geq 0$. For the transformed problem, compute the direction vector $d^k = proj_{null(A^k)}(-\nabla f)$ and the step length α^k. Use these to calculate $\mathbf{y}^{k+1} = \mathbf{y}^k + \beta\alpha^k\mathbf{d}^k$. Finally, transform it back to the original problem by using $\mathbf{x}^{k+1} = T^k\mathbf{y}^{k+1}$.

1. (25 points) Given the quadratic programming problem

$$\begin{aligned} \text{min} \quad & f = x_1^2 + x_1x_2 + 2x_2^2 + x_1 - x_2 \\ \text{subject to} \quad & x_1 - 2x_2 - 2 \geq 0 \\ & -x_1 + x_2 + 1 \geq 0 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

   (a) Is this a convex quadratic programming problem?

   (b) Write down initial tableau for the Lemke's method. You do not need to solve the quadratic programming problem.

2. (25 points) Consider the linear programming problem

$$\begin{aligned} \text{min} \quad & f = -x_1 - 5x_2 \\ \text{subject to} \quad & x_1 + x_2 + x_3 = 3 \\ & x_1,\ x_2,\ x_3 \geq 0 \end{aligned}$$

   (a) Find the value of a such that $\mathbf{x}^0 = [1,\ 1,\ a]^t$ is an interior point for this problem.

   (b) Use the Primal Affine Scaling (PAS) method to compute the next interior point $\mathbf{x}^1$, with $\beta = 0.9$.