Solution to Exam II

1. Notice that

$$Q = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}, \qquad p = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \qquad A = \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}, \qquad b = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

   (a) Compute the eigenvalues of Q, we have $\lambda_1 \approx 1.5858$ and $\lambda_2 \approx 4.4142$. Therefore, the objective function is convex. Hence the quadratic programming problem is convex.

   (b) The initial tableau for the Lemke's method is

```
                 z1           z2           z3           z4            1
        ------------------------------------------------------------
w1 = |      2.0000       1.0000      -1.0000       1.0000       1.0000
w2 = |      1.0000       4.0000       2.0000      -1.0000      -1.0000
w3 = |      1.0000      -2.0000       0.0000       0.0000      -2.0000
w4 = |     -1.0000       1.0000       0.0000       0.0000       1.0000
```

   Or, after adding artificial variable z_0, we have

```
                 z1           z2           z3           z4           z0            1
        -------------------------------------------------------------------------
w1 = |      2.0000       1.0000      -1.0000       1.0000       1.0000       1.0000
w2 = |      1.0000       4.0000       2.0000      -1.0000       1.0000      -1.0000
w3 = |      1.0000      -2.0000       0.0000       0.0000       1.0000      -2.0000
w4 = |     -1.0000       1.0000       0.0000       0.0000       1.0000       1.0000
```

2. (a) An interior point must satisfy all constraints. There fore we have

$$1 + 1 + a = 3,$$

   which implies $a = 3$.

   (b) We have

$$A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, \qquad p = \begin{bmatrix} -1 \\ -5 \\ 0 \end{bmatrix}, \qquad b = \begin{bmatrix} 3 \end{bmatrix}$$

   and

$$x^0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

   Clearly,

$$T^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   Therefore, we have

$$p^0 = T^0 * p = \begin{bmatrix} -1 \\ -5 \\ 0 \end{bmatrix}, \quad A^0 = A * T^0 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Then, by using the formula for d^0 and α^0, we can compute

$$d^0 = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}, \qquad \alpha^0 = 0.5$$

Hence

$$y^1 = y^0 + \beta\alpha^0 d^0 = \begin{bmatrix} 0.55 \\ 2.35 \\ 0.1 \end{bmatrix}$$

Finally,

$$x^1 = T^0 y^1 = \begin{bmatrix} 0.55 \\ 2.35 \\ 0.1 \end{bmatrix}$$