

Math 4553, Homework 4, Due on 4/4/2011

1. (10 points) Determine if the following problems are convex optimization problems or not.

(a) Minimize $f(x, y) = x^2 + y^2 - xy - 24x - 20y$

Subject to
$$\begin{pmatrix} x + 2y \geq 0 \\ x + 2y \leq 9 \\ x + y \leq 8 \\ x + y \geq 0 \end{pmatrix}$$

(b) Maximize $f(x, y) = (x - 2)^2 + (y - 10)^2$

Subject to
$$\begin{pmatrix} x^2 + y^2 = 50 \\ x^2 + y^2 + 2xy - x - y + 20 \geq 0 \\ x \geq 0, y \geq 0 \end{pmatrix}$$

2. (10 points) Consider the quadratic programming problem

$$\begin{aligned} \min \quad & x_1^2 + \frac{1}{4}x_2^2 + 8x_1 - x_2 \\ \text{subject to} \quad & x_1 - 2x_2 \geq -2 \\ & x_1 - x_2 \geq -7 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(a) Show this problem is a convex quadratic programming problem.

(b) Write down the KKT conditions for this problem.

(c) Find the value of c such that

$$\tilde{\mathbf{x}} = \begin{bmatrix} 0 \\ c \end{bmatrix}, \quad \tilde{\mathbf{u}} = \begin{bmatrix} \frac{1}{4} \\ 0 \end{bmatrix}$$

satisfy the KKT conditions.

(d) What is the solution to this quadratic programming problem?