## Math 4553, Homework 3, Due on 3/23/2011

1. (10 points) Use the simplex method to find the shortest route from vertex 1 to vertex 9, where the digraph and distance between cities are given as following:



2. (6 points) Consider the following problem

min 
$$f = -2x_1 + 5x_2 + 4x_3 + x_4 + x_5$$
  
subject to  $2x_1 + 4x_2 + 3x_3 - x_5 = 2$   
 $x_1 - x_2 + 5x_4 + x_5 = 17$   
 $2x_1 + 2x_2 - 2x_3 + 3x_4 = 12$   
 $x_1, x_2, x_3, x_4, x_5 \ge 0$ 

Given  $B = \{x_1, x_4, x_5\}$  and  $N = \{x_2, x_3\}$ , write the corresponding simplex tableau. Is this tableau feasible? If it is feasible, is it optimal? Can you pick the pivot column and pivot row?

3. (4 points) For the following quadratic forms, write them in the form of  $\frac{1}{2}\mathbf{x}^{t}A\mathbf{x}$ :

(a) 
$$f(x_1, x_2, x_3) = 6x_1^2 - 4x_2^2 - 3x_1x_2 + 2x_3^2$$
  
(b)  $f(x_1, x_2, x_3, x_4) = \frac{1}{2}x_1^2 + x_2^2 - 3x_1x_3 + \frac{13}{2}x_3^2 + 2x_1x_4 + x_2x_4 + 7x_4^2$