Math 4553, Solution to Homework 2

1. We first solve the problem using the simplex method. The initial tableau is

	x1	x2	1
y1 =	0.0000	-1.0000	5.0000
y2 =	-1.0000	-1.0000	9.0000
y3 =	-1.0000	2.0000	0.0000
y4 =	1.0000	-1.0000	3.0000
f =	-1.0000	-2.0000	0.0000

The tableau is feasible. Therefore we can start phase II. Perform Jordan exchange of x_2 and y_4, we have

	x1	y4	1
y1 =	-1.0000	1.0000	2.0000
y2 =	-2.0000	1.0000	6.0000
y3 =	1.0000	-2.0000	6.0000
x2 =	1.0000	-1.0000	3.0000
f =	-3.0000	2.0000	-6.0000

Next, perform Jordan exchange of x_1 and y_1,

	y1	y4	1
x1 =	-1.0000	1.0000	2.0000
y2 =	2.0000	-1.0000	2.0000
y3 =	-1.0000	-1.0000	8.0000
x2 =	-1.0000	0.0000	5.0000
f =	3.0000	-1.0000	-12.0000

Finally, perform Jordan exchange of y_2 and y_4,

	y1	y2	1
x1 =	1.0000	-1.0000	4.0000
y4 =	2.0000	-1.0000	2.0000
y3 =	-3.0000	1.0000	6.0000
x2 =	-1.0000	-0.0000	5.0000
f =	1.0000	1.0000	-14.0000

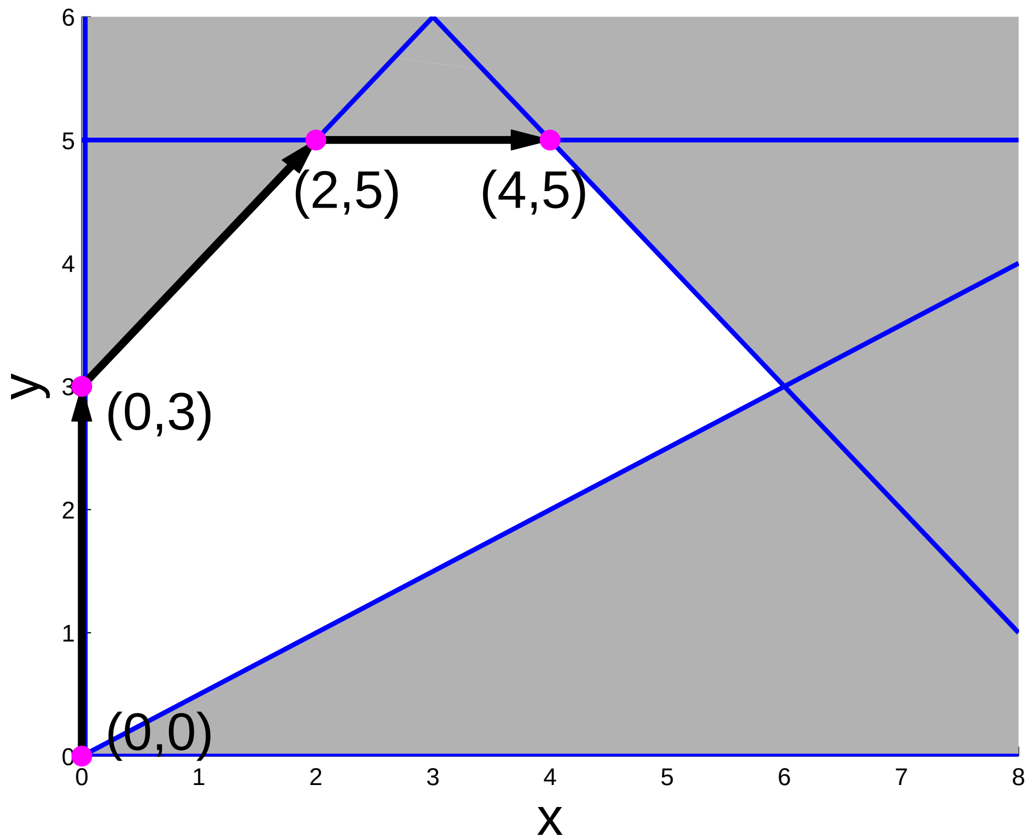
We end up with an optimal tableau. The optimal solution is

$$x_1 = 4, \qquad x_2 = 5$$

$$y_1 = 0, \qquad y_2 = 0, \qquad y_3 = 6, \qquad y_4 = 2$$

$$f = -14$$

By using graphical optimization, we can draw the feasible region of the given problem. Notice that the simplex method given above corresponds to a 'path' in the graph, marked by black arrows.



2. For this problem, the initial tableau is

	x1	x2	x3	x4	1
y1 =	0.0000	1.0000	-2.0000	-1.0000	4.0000
y2 =	2.0000	-1.0000	-1.0000	4.0000	5.0000
y3 =	-1.0000	1.0000	0.0000	-2.0000	3.0000
f =	1.0000	-2.0000	-4.0000	4.0000	0.0000

It is feasible. So we can start phase II. Exchange y_1 with x_3, we have

	x1	x2	y1	x4	1
x3 =	0.0000	0.5000	-0.5000	-0.5000	2.0000
y2 =	2.0000	-1.5000	0.5000	4.5000	3.0000
y3 =	-1.0000	1.0000	-0.0000	-2.0000	3.0000
f =	1.0000	-4.0000	2.0000	6.0000	-8.0000

Exchange y_2 with x_2,

	x1	y2	y1	x4	1
x3 =	0.6667	-0.3333	-0.3333	1.0000	3.0000
x2 =	1.3333	-0.6667	0.3333	3.0000	2.0000
y3 =	0.3333	-0.6667	0.3333	1.0000	5.0000
f =	-4.3333	2.6667	0.6667	-6.0000	-16.0000

No pivot row can be chosen at this stage. Therefore we know the problem is unbounded. By setting $x_4 = \lambda$, we have

$$f = -6\lambda - 16$$

To make $f = -415$, we only need to set $\lambda = 66.5$. That is

$$x_1 = 0, \qquad x_2 = 3\lambda + 2 = 201.5, \qquad x_3 = \lambda + 3 = 69.5, \qquad x_4 = \lambda = 66.5$$

$$y_1 = 0, \qquad y_2 = 0, \qquad y_3 = \lambda + 5 = 71.5$$

$$f = -6\lambda - 16 = -415$$

Alternatively, one can set $x_1 = \lambda$ and calculate the feasible point.

3. the initial tableau is

	x1	x2	1
y1 =	-1.0000	-1.0000	2.0000
y2 =	2.0000	2.0000	-10.0000
f =	-3.0000	1.0000	0.0000

It is not feasible. We need to go through phase I. First, add artificial variable x_0 and new objective function f_0,

	x1	x2	x0	1
y1 =	-1.0000	-1.0000	0.0000	2.0000
y2 =	2.0000	2.0000	1.0000	-10.0000
f =	-3.0000	1.0000	0.0000	0.0000
f0 =	0.0000	0.0000	1.0000	0.0000

Perform a special pivot of x_0 and y_2,

	x1	x2	y2	1
y1 =	-1.0000	-1.0000	0.0000	2.0000
x0 =	-2.0000	-2.0000	1.0000	10.0000
f =	-3.0000	1.0000	0.0000	0.0000
f0 =	-2.0000	-2.0000	1.0000	10.0000

To minimize f_0, exchange x_1 with y_1,

	y1	x2	y2	1
x1 =	-1.0000	-1.0000	0.0000	2.0000
x0 =	2.0000	0.0000	1.0000	6.0000
f =	3.0000	4.0000	0.0000	-6.0000
f0 =	2.0000	0.0000	1.0000	6.0000

The current tableau is optimal with respect to f_0. However, the minimal value of f_0 is 6. Therefore, we know the original problem is infeasible.

4. The initial tableau is

	x1	x2	x3	x4	1
y1 =	-1.0000	-3.0000	0.0000	-1.0000	4.0000
y2 =	-2.0000	-1.0000	0.0000	0.0000	3.0000
y3 =	0.0000	-1.0000	-4.0000	-1.0000	3.0000
y4 =	1.0000	1.0000	2.0000	0.0000	-1.0000
y5 =	-1.0000	1.0000	4.0000	0.0000	-1.0000
f =	-2.0000	-4.0000	-1.0000	-1.0000	0.0000

It is not feasible, so we add artificial variable x_0 and new objective function f_0,

	x1	x2	x3	x4	x0	1
y1 =	-1.0000	-3.0000	0.0000	-1.0000	0.0000	4.0000
y2 =	-2.0000	-1.0000	0.0000	0.0000	0.0000	3.0000
y3 =	0.0000	-1.0000	-4.0000	-1.0000	0.0000	3.0000
y4 =	1.0000	1.0000	2.0000	0.0000	1.0000	-1.0000
y5 =	-1.0000	1.0000	4.0000	0.0000	1.0000	-1.0000
f =	-2.0000	-4.0000	-1.0000	-1.0000	0.0000	0.0000
f0 =	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000

Perform a special pivot with x_0 and y_4,

	x1	x2	x3	x4	y4	1
y1 =	-1.0000	-3.0000	0.0000	-1.0000	0.0000	4.0000
y2 =	-2.0000	-1.0000	0.0000	0.0000	0.0000	3.0000
y3 =	0.0000	-1.0000	-4.0000	-1.0000	0.0000	3.0000
x0 =	-1.0000	-1.0000	-2.0000	-0.0000	1.0000	1.0000
y5 =	-2.0000	0.0000	2.0000	0.0000	1.0000	0.0000
f =	-2.0000	-4.0000	-1.0000	-1.0000	0.0000	0.0000
f0 =	-1.0000	-1.0000	-2.0000	0.0000	1.0000	1.0000

To minimize f_0, exchange x_3 and x_0,

	x1	x2	x0	x4	y4	1
y1 =	-1.0000	-3.0000	-0.0000	-1.0000	0.0000	4.0000
y2 =	-2.0000	-1.0000	-0.0000	0.0000	0.0000	3.0000
y3 =	2.0000	1.0000	2.0000	-1.0000	-2.0000	1.0000
x3 =	-0.5000	-0.5000	-0.5000	-0.0000	0.5000	0.5000
y5 =	-3.0000	-1.0000	-1.0000	0.0000	2.0000	1.0000
f =	-1.5000	-3.5000	0.5000	-1.0000	-0.5000	-0.5000
f0 =	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000

Notice the current tableau is optimal with respect to f_0 and the minimum value of f_0 is 0. This means we have located a basic feasible solution (vertex) for the original problem. Hence we can delete x_0 column and f_0 row, then start phase II.

	x1	x2	x4	y4	1
y1 =	-1.0000	-3.0000	-1.0000	0.0000	4.0000
y2 =	-2.0000	-1.0000	0.0000	0.0000	3.0000
y3 =	2.0000	1.0000	-1.0000	-2.0000	1.0000
x3 =	-0.5000	-0.5000	-0.0000	0.5000	0.5000
y5 =	-3.0000	-1.0000	0.0000	2.0000	1.0000
f =	-1.5000	-3.5000	-1.0000	-0.5000	-0.5000

Exchange x_2 and x_3,

	x1	x3	x4	y4	1
y1 =	2.0000	6.0000	-1.0000	-3.0000	1.0000
y2 =	-1.0000	2.0000	0.0000	-1.0000	2.0000
y3 =	1.0000	-2.0000	-1.0000	-1.0000	2.0000
x2 =	-1.0000	-2.0000	-0.0000	1.0000	1.0000
y5 =	-2.0000	2.0000	0.0000	1.0000	0.0000
f =	2.0000	7.0000	-1.0000	-4.0000	-4.0000

Exchange y_1 and y_4,

	x1	x3	x4	y1	1
y4 =	0.6667	2.0000	-0.3333	-0.3333	0.3333
y2 =	-1.6667	0.0000	0.3333	0.3333	1.6667
y3 =	0.3333	-4.0000	-0.6667	0.3333	1.6667
x2 =	-0.3333	0.0000	-0.3333	-0.3333	1.3333
y5 =	-1.3333	4.0000	-0.3333	-0.3333	0.3333
f =	-0.6667	-1.0000	0.3333	1.3333	-5.3333

Exchange x_3 and y_3,

	x1	y3	x4	y1	1
y4 =	0.8333	-0.5000	-0.6667	-0.1667	1.1667
y2 =	-1.6667	-0.0000	0.3333	0.3333	1.6667
x3 =	0.0833	-0.2500	-0.1667	0.0833	0.4167
x2 =	-0.3333	-0.0000	-0.3333	-0.3333	1.3333
y5 =	-1.0000	-1.0000	-1.0000	0.0000	2.0000
f =	-0.7500	0.2500	0.5000	1.2500	-5.7500

Exchange x_1 and y_2,

	y2	y3	x4	y1	1
y4 =	-0.5000	-0.5000	-0.5000	0.0000	2.0000
x1 =	-0.6000	-0.0000	0.2000	0.2000	1.0000
x3 =	-0.0500	-0.2500	-0.1500	0.1000	0.5000
x2 =	0.2000	-0.0000	-0.4000	-0.4000	1.0000
y5 =	0.6000	-1.0000	-1.2000	-0.2000	1.0000
f =	0.4500	0.2500	0.3500	1.1000	-6.5000

We have the optimal tableau. The optimal solution is

$$x_1 = 1, \quad x_2 = 1, \quad x_3 = 0.5, \quad x_4 = 0$$

$$y_1 = 0, \quad y_2 = 0, \quad y_3 = 0, \quad y_4 = 2, \quad y_5 = 1$$

$$f = -6.5$$