Formula:

- When using the simplex method for solving linear programming problems, we have

$$\begin{array}{c|c|c|}
 & x_N & 1 \\ \hline
x_B & -A_B^{-1}A_N & A_B^{-1}b \\ \hline
f & p_N^t - p_B^t A_B^{-1} A_N & p_B^t A_B^{-1} b \\ \hline
\end{array}$$

- Given a vector $\mathbf{v}$ and a matrix A, we have

$$proj_{null(A)}\mathbf{v} = \left(I - A^T(AA^T)^{-1}A\right)\mathbf{v}$$

- (PAS algorithm) Given the k-th interior point $\mathbf{x}^k$, compute the affine transformation $\mathbf{x}^k = T^k\mathbf{y}^k$, where $\mathbf{y}^k = (1, \ldots, 1)^t$. Use this affine transformation to rewrite the linear programming problem into *minimize* $f = \mathbf{p}^k\mathbf{y}$, *subject to* $A^k\mathbf{y} = \mathbf{b}$, $\mathbf{y} \geq 0$. For the transformed problem, compute the direction vector $d^k = proj_{null(A^k)}(-\nabla f)$ and the step length α^k. Use these to calculate $\mathbf{y}^{k+1} = \mathbf{y}^k + \beta\alpha^k\mathbf{d}^k$. Finally, transform it back to the original problem by using $\mathbf{x}^{k+1} = T^k\mathbf{y}^{k+1}$.

- (Primal-dual interior point method) Given $\mathbf{x}^k$, $\mathbf{s}^k$, $\mathbf{u}^k$, compute $\mu^k = \frac{\mathbf{x}^k \cdot \mathbf{s}^k}{n(k+1)}$, $D = XS^{-1}$, $\mathbf{r}_a = -A\mathbf{x}^k + \mathbf{b}$, $\mathbf{r}_b = -A^t\mathbf{u}^k - \mathbf{s}^k + \mathbf{p}$, $\mathbf{r}_c = -XS\mathbf{e} + \mu\mathbf{e}$. Then compute $\mathbf{d}_u = (ADA^T)^{-1}(\mathbf{r}_a + AD\mathbf{r}_b - AS^{-1}\mathbf{r}_c)$, $\mathbf{d}_s = \mathbf{r}_b - A^t\mathbf{d}_u$, $\mathbf{d}_x = S^{-1}\mathbf{r}_c - D\mathbf{d}_s$. Use the ratio test to compute α_x, α_s, and α_u. Finally, $\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha_x\mathbf{d}_x$, $\mathbf{s}^{k+1} = \mathbf{s}^k + \alpha_s\mathbf{d}_s$, $\mathbf{u}^{k+1} = \mathbf{u}^k + \alpha_u\mathbf{d}_u$.

- The Lagrange function for problem

$$\begin{array}{rl}
\text{min} & f(\mathbf{x}) \\
\text{subject to} & g_1(\mathbf{x}) \geq b_1 \\
 & \cdots \\
 & g_m(\mathbf{x}) \geq b_m
\end{array}$$

  is

$$L(\mathbf{x},\, \mathbf{u}) = f(\mathbf{x}) - u_1(g_1(\mathbf{x}) - b_1) - \cdots - u_m(g_m(\mathbf{x}) - b_m)$$

  where $\mathbf{u} \geq 0$.

- (Active set algorithm) Given $\mathbf{x}^k$, use the Lagrange function to check whether it is an optimal solution or not. If $\mathbf{x}^k$ is not an optimal solution, remove the constraint that corresponds to to the most negative Lagrange multiplier from the active set. Then, compute $\mathbf{y}_k$ satisfying

$$\begin{array}{rl}
\text{min} & f(\mathbf{x}^k + \mathbf{y^k}) \\
\text{subject to} & (\mathbf{x}^k + \mathbf{y^k} \text{ satisfies all the rest of active constrains})
\end{array}$$

  Set $\mathbf{d}_k = \mathbf{y}_k$. Next, compute $\alpha^k = min(1,\, \bar{\alpha})$, where $\bar{\alpha}$ is the largest number which guarantees that $\mathbf{x}^k + \bar{\alpha}\mathbf{d}^k$ satisfies all inactive constraints. Set $\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha^k\mathbf{d}^k$.