Reduced Row Echelon Form

Example

Find the Reduced Row Echelon of the given matrix A:

In[1]:=
```
A = {{2, 4, 6, -6, 0, 0, 0},
   {3, -5, 3, -3, 1, 0, 0}, {4, 9, -4, 4, 0, -1, 0}, {0, 1, 0, 0, 0, 0, 1}};
A //
 MatrixForm
```

Out[2]//MatrixForm=

$$\begin{pmatrix} 2 & 4 & 6 & -6 & 0 & 0 & 0 \\ 3 & -5 & 3 & -3 & 1 & 0 & 0 \\ 4 & 9 & -4 & 4 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[3]:=
```
ReducedA = RowReduce[A];
ReducedA // MatrixForm
```

Out[4]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -\frac{3}{16} & -\frac{35}{16} \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & \frac{1}{16} & \frac{1}{16} \\ 0 & 0 & 0 & 0 & 1 & \frac{3}{8} & \frac{91}{8} \end{pmatrix}$$

In[5]:=
```
MatrixRank[A]
```

Out[5]=
```
4
```

In[6]:=
```
NullSpace[A]
```

Out[6]=
```
{{35, -16, -1, 0, -182, 0, 16}, {3, 0, -1, 0, -6, 16, 0}, {0, 0, 1, 1, 0, 0, 0}}
```

In case that we would like to compute the Reduced Row Echelon form of the augmented matrix for Ax = b:

In[7]:=
```
b = {{11}, {11}, {12}, {4}};
b // MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 11 \\ 11 \\ 12 \\ 4 \end{pmatrix}$$

In[9]:=
```
AugmentedMatrix = Join[A, b, 2];
AugmentedMatrix // MatrixForm
```

Out[10]//MatrixForm=

$$\begin{pmatrix} 2 & 4 & 6 & -6 & 0 & 0 & 0 & 11 \\ 3 & -5 & 3 & -3 & 1 & 0 & 0 & 11 \\ 4 & 9 & -4 & 4 & 0 & -1 & 0 & 12 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 4 \end{pmatrix}$$

In[11]:=
```
RowReduce[AugmentedMatrix] // MatrixForm
```

Out[11]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -\frac{3}{16} & -\frac{35}{16} & -\frac{41}{8} \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & -1 & 0 & \frac{1}{16} & \frac{1}{16} & \frac{7}{8} \\ 0 & 0 & 0 & 0 & 1 & \frac{3}{8} & \frac{91}{8} & \frac{175}{4} \end{pmatrix}$$