Math 4513, Homework 2, Due on 10/3/2014

1. (8 points) The following sequence

$$p_0 = 0.75, \qquad p_n = \left(\frac{e^{p_{n-1}}}{3}\right)^{0.5} \quad \text{for } n = 1, 2, 3, \cdots$$

   is linearly convergent.

   (a) Which number does p_n converge to? Write down your estimation with at least 3 significant digits.

   (b) Compute the first 5 terms of $\{\hat{p}_n\}$ using the Aitken's acceleration.

   (c) Compare $\{p_n\}$ and $\{\hat{p}_n\}$. Do they converge to the same number? Which one converges faster?

2. (6 points) Use Horner's method to compute $P(4)$ and $P'(4)$ where

$$P(x) = 3x^5 + x^4 - 2x^3 - 5x^2 + 12x - 5$$

3. (6 points) Given a function $f(x) = \sqrt{1+x}$ and points $x_0 = 0$, $x_1 = 0.6$, $x_2 = 0.9$. Denote $y_i = f(x_i)$ for $i = 0, 1, 2$. Construct a Lagrange interpolation $P(x)$ using points (x_i, y_i) for $i = 0, 1, 2$. Use $P(x)$ to estimate $f(0.45)$.