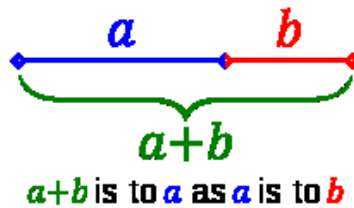


Math 4513, Homework 1, Due on 9/19/2014

1. (6 points) Find the rate of convergence of the following function as $h \rightarrow 0$

$$\lim_{h \rightarrow 0} \frac{\sin h - h \cos h}{h} = 0$$

2. (8 points) Here's another way to compute π : pick a random number $p_0 \in [2, 4]$ and then start the iteration $p_n = p_{n-1} + \sin p_{n-1}$ for $n = 1, 2, 3, \dots$. Write a Matlab program and test it with a few choices of p_0 . You will observe that the iteration converges to π quickly. Try to explain why it converges using the theorems we've covered in class. This includes identifying which theorem to use and showing that the iteration satisfies all conditions of the theorem.
3. (6 points) The golden ratio has long been thought as aesthetically pleasing. It is described in the following graph (from wikipedia):



which means

$$\frac{a+b}{a} = \frac{a}{b}$$

Set $x = \frac{a}{b}$, the above equation becomes

$$\frac{(a+b)/b}{a/b} = \frac{a}{b} \implies \frac{x+1}{x} = x \implies x^2 - x - 1 = 0$$

The positive root of $x^2 - x - 1 = 0$ is the famous golden ratio. (Of course you should know how to find this root using the quadratic formula)

In Matlab, use the Newton's method to compute the golden ratio to at least 10 significant digits.