Math 4513, Homework 3, Due on 10/12/2012

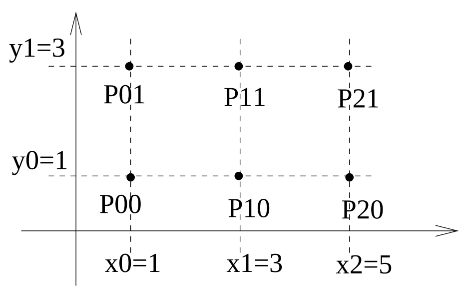
1. (6 points) Given $x_0 = 0$, $x_1 = 2$, $x_2 = 3$, $x_3 = 5$, $x_4 = 6$ and $f(x) = x^3$, compute $f[x_0, x_1, x_2, x_3]$ and $f[x_0, x_1, x_2, x_3, x_4]$. Explain your result using the following property of Newton's divided difference:
$$f[x_0, x_1, \ldots, x_n] = \frac{f^n(\xi)}{n!}.$$

2. (6 points) It is known that function $f(x)$ satisfies

x	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
f(x)	0.0385	0.0588	0.1	0.2	0.5	1	0.5	0.2	0.1	0.0588	0.0385

Use Matlab to compute the Lagrange interpolation of the above data. Draw the graph of your interpolation polynomial.

3. (8 points) Using the idea covered in class, we can generate two-dimensional or higher-dimensional Lagrange interpolations. Here we only consider a simple two-dimensional example. Given 6 points as following



where the coordinates of these 6 points are

$$P01 = (1,3) \qquad P11 = (3,3) \qquad P21 = (5,3)$$
$$P00 = (1,1) \qquad P10 = (3,1) \qquad P20 = (5,1)$$

Here for simplicity, denote $x_0 = 1$, $x_1 = 3$, $x_2 = 5$ and $y_0 = 1$, $y_1 = 3$. We would like to find a polynomial $P(x,y)$ which has degree 2 in x and degree 1 in y, that is

$$P(x,y) = a + bx + cy + dxy + ex^2 + fx^2y, \qquad \text{where } a,b,c,d,e,f \text{ are parameters,}$$

and satisfy

$$P(x_0,y_1) = 7 \qquad P(x_1,y_1) = 19 \qquad P(x_2,y_1) = 23$$
$$P(x_0,y_0) = 3 \qquad P(x_1,y_0) = 3 \qquad P(x_2,y_0) = -5$$

The easiest way to calculate $P(x,y)$ is to use the following formula:

$$P(x,y) = \sum_{i=0}^{2}\sum_{j=0}^{1} P(x_i,y_j)L_{2,i}(x)L_{1,j}(y),$$

where $L_{2,i}(x)$ and $L_{1,j}(y)$ are the Lagrange interpolation polynomial we have defined in class. Use this formula to compute $P(x,y)$. Find the value of a,b,c,d,e,f.