Math 4513, Homework 1, Due on 9/19/2012

1. (10 points) Find the rate of convergence of the following function as $h \to 0$

$$\lim_{h \to 0} \frac{1 - e^h}{h} = -1$$

2. (10 points) Nowadays, you can easily find that $\sqrt{2} \approx 1.414213562373095\ldots$ using a calculator. Have you ever wondered how a calculator does this? Or more general, how to compute the principal square root (or even the nth root) of any nonnegative real number?

   Indeed, there are many different numerical algorithms developed for this purpose. For example, pocket calculators compute squareroots using exponential and logarithmic functions, since their hardware is designed to do these two operations very fast. For more information, you may check

   `http://en.wikipedia.org/wiki/Methods_of_computing_square_roots`

   Here, we would like to compute $\sqrt{2}$ using two algorithms introduced in our class, the bisection method and the Newton's method. Notice that to compute $\sqrt{2}$ is the same as to find the positive root of

$$x^2 - 2 = 0.$$

   Now, use the bisection method and the Newton's method to solve this root-finding problem with $TOL = 10^{-8}$. You may specify the initial interval $[a, b]$ in the bisection method, or the initial guess p_0 in the Newton's method, as you like, as long as the values you choose are reasonable. Or you can play with several different choices of initial values to see if they will make any difference. Report your code and outputs.

   The following part will not count towards the total credit of this problem. If you are interested, think about whether the fixed-point iteration can be applied here or not? Why?