Math 4513, Homework 4, Due on 11/11/2011

1. (8 points) Consider $\int_a^b f(x)\,dx$. Divide $[a,b]$ into n subintervals where n is a multiple of 3. In other words, n can be written as $3m$ where m is an integer. Let $h = \frac{b-a}{n}$ and denote $x_i = a + ih$. There are n subintervals in total and they can be further divided into m groups, each group containing 3 consecutive subintervals. These subgroups can be expressed as (x_{3i}, x_{3i+3}) for $i = 0, 1, \ldots, m-1$.

   Recall that a 2-point open Newton-Cotes formula on the interval $(x_0,\, x_3)$ is

   $$\int_{x_0}^{x_3} f(x)dx \approx \frac{3h}{2}\Big(f(x_1) + f(x_2)\Big)$$

   (a) Derive a composite numerical integration formula which approximates $\int_a^b f(x)\,dx$ using the 2-point open Newton-Cotes formula on each group of 3 consecutive subintervals.

   (b) Apply your formula to estimate $\int_{-1}^2 \frac{x}{x^2+4}\,dx$ with $h = 0.1$. What is the eror between the approximate integral and the exact value of the integral? (Use `format long e` in Matlab to get 15 digits of you answer.)

2. (6 points) Consider $\int_0^{\pi/2} x^2 \sin x\,dx$. Compute its exact value. Then approximate the integral using Gaussian quadratures with $n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$. Find the error for each n. (Use `format long e` in Matlab to get 15 digits of you answer.)

3. (6 points) Use the Euler's method to approximate the solution of

   $$y' = \frac{1+t}{1+y}, \qquad 1 \le t \le 2, \qquad y(1) = 2, \qquad h = 0.5$$