Math 4513, Homework 2, Due on 10/7/2011

1. (6 points) Theorem 2.14 has been given without proof in class. It states that

   Theorem 2.14 Suppose that $\{p_n\}_{n=0}^{\infty}$ is a sequence that converges linearly to the limit p and that

   $$\lim_{n\to\infty} \frac{p_{n+1}-p}{p_n-p} = \lambda < 1.$$

   The the Aitken's sequence $\{\hat{p}_n\}_0^{\infty}$ defined by

   $$\hat{p}_n = p_n - \frac{(p_{n+1}-p_n)^2}{p_{n+2}-2p_{n+1}+p_n}$$

   converges to p faster than $\{p_n\}_{n=0}^{\infty}$ in the sense that

   $$\lim_{n\to\infty} \frac{\hat{p}_n-p}{p_n-p} = 0.$$

   Prove the theorem. Hint: First show that

   $$\hat{p}_n - p = (p_n-p) - \frac{((p_{n+1}-p)-(p_n-p))^2}{(p_{n+2}-p)-2(p_{n+1}-p)+(p_n-p)}$$

   Define $\lambda_n = \frac{p_{n+1}-p}{p_n-p}$. Rewrite $\frac{\hat{p}_n-p}{p_n-p}$ in terms of λ_n and λ_{n+1} and take the limit for $n \to \infty$.

2. (7 points) Given

   $$P(x) = x^5 - x^4 + 2x^3 - 3x^2 + x - 4.$$

   Use Horner's method to evaluate $P(2)$ and $P'(2)$. You can either write a program to solve it, or compute it by hand. If you do it by hand, please give the details of using the Horner's method.

3. (7 points) Compute the 4th degree Lagrange polynomial using points

   $$(-1, 15), \qquad (0, 5), \qquad (1, 3), \qquad (2, 9), \qquad (3, 47).$$

   You can use any method you prefer. Write the details. The polynomial in your final answer should be written in the form of

   $$P(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0.$$