Math 4513, Homework 1, Due on 9/12/2011

1. It is known that
$$\lim_{h\to 0} \frac{\sin h - h\cos h}{h} = 0.$$
   (a) Use the Taylor expansion to determine its rate of convergence in $O(h^n)$.

   (b) Compute the value of $f(h) = \frac{\sin h - h\cos h}{h}$ for $h = \frac{1}{2}, \frac{1}{2^2}, \ldots, \frac{1}{2^{20}}$. Then plot $f(h)$ vs. h in Matlab/Octave.

   (c) Use the Matlab/Octave command `polyfit` to find the polynomial that best fits the curve $f(h)$. Type `help polyfit` in Matlab/Octave for more information and how to use this function.

   (d) Plot $f(h)$ and its best fitting polynomial in the same graph.

2. Consider the equation
$$x = rx(1-x),$$
where $r > 0$. The equation has two roots, 0 and $\frac{r-1}{r}$. In this problem, set $TOL = 10^{-8}$ and $N = 100$.

   (a) Write a program to solve the problem using the bisection methods, and compare the approximate solution with the exact solution. Set the initial interval to be $[a, b] = [0.5,\ 1]$, and test the following set of values for r:
$$r = 2.6,\ 3.2,\ 3.5,\ 3.8,\ 4.2$$

   (b) Now try to solve the same problem using the fixed point iteration. Set the initial value to be $x_0 = 0.5$, and test the following set of values for r:
$$r = 0.5,\ 1.5,\ 2.6,\ 3.2,\ 3.5,\ 3.8,\ 4.2$$

   What you should observe is the famous chaotic behavior of the logistic map. Draw the graph of vector $(x_0, x_1, ..., x_N)$, you should see the following:

   i. When $0 < r < 1$, the fixed point iteration converges to 0.

   ii. When $1 < r < 2$, the fixed point iteration converges to $\frac{r-1}{r}$ quickly.

   iii. When $2 < r < 3$, the fixed point iteration converges to $\frac{r-1}{r}$, but with some fluctuation in the beginning.

   iv. When $3 < r < 1 + \sqrt{6} \approx 3.45$, the fixed point iteration oscillates between 2 values (for most initial values).

   v. When $3.45 < r < 3.54$ (approximately), the fixed point iteration oscillates among 4 values (for most initial values).

   vi. As r increases beyond 3.54, the fixed point iteration oscillates among 8 values, then 16, 32, ...

   vii. As r reachs 3.57, the oscillation no longer has a finite orbit. The chaos starts.

   viii. For $r > 4$, the fixed point iteration diverges to ∞ (for most initial values).