Math 3013, Exam 3, Apr. 19, 2011

Name: _____

Score:

The total is 50 points. Problem 1-3 are worth 4 points each. Calculators are NOT allowed.

1. ( ) Which of the following is NOT a subspace of the vector space $M_{2\times 3}$, of all 2×3 matrices. The set of all matrices of the form:

   (a) $\begin{bmatrix} a & a+c & c \\ b & 0 & 0 \end{bmatrix}$;

   (b) $\begin{bmatrix} a & b & c \\ b & 0 & 0 \end{bmatrix}$;

   (c) $\begin{bmatrix} a & b & c \\ d & e & 0 \end{bmatrix}$;

   (d) $\begin{bmatrix} a & a+c & c \\ 2c+1 & 0 & 0 \end{bmatrix}$;

   (e) $\begin{bmatrix} a & b & c \\ d & d-c & 0 \end{bmatrix}$.

2. ( ) Let V be a vector space and $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be vectors in V. Which of the following is NOT true.

   (a) If $\mathbf{u} \perp \mathbf{v}$ and $\mathbf{u} \perp \mathbf{w}$, then $\mathbf{u} \perp (\mathbf{v} + \mathbf{w})$.

   (b) If $\mathbf{u} \perp \mathbf{v}$ and $\mathbf{v} \perp \mathbf{w}$, then $\mathbf{u} \perp \mathbf{w}$.

   (c) If $\mathbf{u} \perp \mathbf{v}$, then $proj_{\mathbf{u}}\mathbf{v} = \mathbf{0}$

   (d) If $\mathbf{u} \perp \mathbf{v}$, then $perp_{\mathbf{u}}\mathbf{v} = \mathbf{v}$

   (e) $\mathbf{0}$ is orthogonal to all other vectors in V.

3. ( ) Let $\{\mathbf{u}, \mathbf{v}\}$ be an orthonormal set of vectors, calculate $(\mathbf{u} + 3\mathbf{v}) \cdot (5\mathbf{u} - 2\mathbf{v})$.
   $(a)\ 5 \qquad (b)\ -6 \qquad (c)\ -1 \qquad (d)\ -2 \qquad (e)\ 11$

4. (8 points) Determine whether A is diagonalizable and, if so, find diagonal matrix D, and invertible matrix P such that $A = PDP^{-1}$. (You do not need to find P^{-1}).

$$A = \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}$$

5. (10 points) Let $S = \{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix} \}$ be a basis for a vector space W.

(1) Find an orthonormal basis for the space W.

(2) Find a basis for $W^{\perp}$.

6. (10 points) Find the QR factorization of

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}$$

7. (10 points) Sketch the graph of $3x^2 - 2xy + 3y^2 = 8$.