Math 3013, Exam 2, Mar. 8, 2011

Name: ____________________

Score:

The total is 50 points. Problem 1-3 are worth 4 points each.

1. (   ) Let A be an $n \times n$ invertible matrix, which of the following statement is **not** true:

   (a) A^T is symmetric.

   (b) A^{-1} exists.

   (c) $A\mathbf{x} = \mathbf{b}$ has a unique solution.

   (d) $A\mathbf{x} = \mathbf{0}$ has only trivial solution.

   (e) A is row equivalent to I_n.

2. (   ) Which one of the following is **not** true:

   (a) If A is a symmetric matrix, then A^T is also a symmetric matrix.

   (b) If A is an $n \times n$ matrix, then $A + A^T$ is symmetric.

   (c) If A and B are $n \times n$ symmetric matrices, then AB is symmetric.

   (d) If A and B are $n \times n$ invertible matrices, then AB is also invertible.

   (e) If A is an $n \times n$ invertible matrix and c is a non-zero number, then cA is also invertible.

3. (   ) Compute $det \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 4 \end{bmatrix}$.

   $(a)\ -2 \qquad (b)\ -1 \qquad (c)\ 4 \qquad (d)\ 5 \qquad (e)\ 6$

4. (6 points) E is a 3×3 matrix of the form

$$E = \begin{bmatrix} 1 & 8 & 3 \\ x & y & z \\ -3 & 7 & 2 \end{bmatrix}.$$

Given $det(E) = 5$. Compute the determinant of

$$F = \begin{bmatrix} x & y & z \\ 1 & 8 & 3 \\ -3+4x & 7+4y & 2+4z \end{bmatrix}.$$

5. (8 points) Let $L : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation satisfying $L(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $L(\mathbf{e}_2) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $L(\mathbf{e}_3) = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$, where $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are the standard basis for $\mathbb{R}^3$. Calculate $L(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix})$.

6. (8 points) Find the eigenvalues and corresponding eigenvectors for

$$A = \begin{bmatrix} 0 & 2 \\ 8 & 6 \end{bmatrix}$$

7. (8 points) Determine whether the following matrix is invertible or not. If it is invertible, find its inverse.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 5 & 1 \\ 2 & 3 & 0 \end{bmatrix}$$

8. (8 points) Find basis for $row(A)$, $col(A)$ and $null(A)$, where

$$A = \begin{bmatrix} 1 & 3 & 4 & 1 & 4 \\ 2 & 3 & 2 & 2 & 5 \\ 0 & 2 & 4 & 0 & 2 \\ 1 & 2 & 2 & 1 & 3 \end{bmatrix}$$