Math 3013, Exam I, Feb. 10, 2011

Name: _____

Score:

The total is 50 points. Problem 1-3 are worth 4 points each.

1. ( ) Which one of the following is **not** a reduced row echelon from:
   (a) $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$ (c) $\begin{bmatrix}1 & 1 & 0 & 1\\ 0 & 0 & 1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$
   (e) $\begin{bmatrix}1 & 0 & 0 & 0.9999\\ 0 & 1 & 0 & 1.9999\\ 0 & 0 & 1 & 2.9999\end{bmatrix}$

2. ( ) Find out the values for a, b and c that will make the following matrix symmetric:

$$\begin{bmatrix}1 & 1 & a+1\\ 1 & c & 2b\\ 2 & 4 & 1\end{bmatrix}$$

   (a) a=1/2, b=2, c=1

   (b) a=1, b=2, c=any number

   (c) a=1/2, b=1, c=any number

   (d) a=1, b=2, c=1

   (e) a=1/2, b=1, c=1

3. ( ) Let A be a 3×2 matrix, B be a 2×2 matrix and C be a 3×3 matrix, which one of the following does **not** exist:
   (a) CAB^T (b) C^TAB (c) CAB (d) BA^TC (e) BAC

4. (6 points) Given point $P = (0, 1, 0)$ and a vector $\mathbf{v} = [3, 2, 1]$.

   (a) Write the equation of a line passing through P with direction vector $\mathbf{v}$.

   (b) Write the equation of a plane passing through P with normal vector $\mathbf{v}$.

5. (8 points) Calculate the following:

   (a) Let $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, find A^4;

   (b) Let $B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, find B^3.

6. (8 points) In the following system, determine all values of a for which the system has

   (a) No solution;

   (b) A unique solution;

   (c) Infinitely many solutions.

$$\begin{cases} x + y = 3, \\ x + (a^2 - 8)y = a. \end{cases}$$

7. (8 points) Solve the following linear system:

$$\begin{cases} 2x_1 - x_2 = 1, \\ -x_1 + 2x_2 - x_3 = 1, \\ -x_2 + 2x_3 = 1. \end{cases}$$

8. (8 points) Determine whether the following vectors are linearly dependent or not. If they are linearly dependent, find a dependence relationship among the vectors.

$$\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \qquad \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \qquad \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$