Math 3013 Linear Algebra, Fall 2012
Quiz 5

Nov. 19, 2012

1. It is known that matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ has two eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 3$, with corrsponding eigenvectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

Solution Matrix A has two linearly independent eigenvectors and hence it is diagonalizable. Let

$$P = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \qquad \text{and} \qquad D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix},$$

then $P^{-1}AP = D$.

Note, an alternative solution is

$$P = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \text{and} \qquad D = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix},$$

2. Is the following matrix orthogonal? Find its inverse.

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Solution Notice that

$$\begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = 0$$

$$\left\| \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} \right\| = 1$$

$$\left\| \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \right\| = 1$$

Clearly, the columns of A form an orthonormal set of vectors. Thus A is an orthogonal matrix.

For orthogonal matrices, we know that $A^{-1} = A^t$. Hence

$$A^{-1} = A^t = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$