

Math 3013 Linear Algebra, Fall 2012
Quiz 4

Oct. 31, 2012

1. It is known that -2 is an eigenvalue of the matrix $A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$. Find an eigenvector corresponding to this eigenvalue.

Solution Use the formula $(A - \lambda I)\mathbf{x} = \mathbf{0}$, we have

$$(A - (-2)I)\mathbf{x} = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Solve this homogeneous equation, one ends up with

$$\mathbf{x} = s \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

For example, if one chooses $s = 2$, then we have an eigenvector $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

2. Compute the determinant of

$$A = \begin{bmatrix} 3 & -2 & 0 & 1 \\ 1 & 3 & 0 & -1 \\ 0 & 2 & 2 & 4 \\ 3 & 1 & 0 & 0 \end{bmatrix}$$

Solution Using the cofactor expansion along the third column, one has

$$\begin{aligned} \det(A) &= 2 \begin{vmatrix} 3 & -2 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & 0 \end{vmatrix} \\ &= 2 \left(3 \times \begin{vmatrix} -2 & 1 \\ 3 & -1 \end{vmatrix} - 1 \times \begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} \right) \\ &= 2 \left(3 \times (-1) - 1 \times (-4) \right) \\ &= 2 \end{aligned}$$