Math 3013 Linear Algebra, Fall 2012
Quiz 1

Sept. 7, 2012

1. (4 points) Give the vector equation of the line passing through

$$P = (1, -2, 4), \qquad Q = (3, 0, -3)$$

Solution Subtract the coordinates of these two points shall give us a direction vector for the line, that is

$$\mathbf{v} = (1, -2, 4) - (3, 0, -3) = (-2, -2, 7)$$

Now, we can write the vector equation of the line as

$$\mathbf{x} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} + t \begin{bmatrix} -2 \\ -2 \\ 7 \end{bmatrix}$$

where t is a parameter.

(Remark: there may be different type of solutions, depending on how you pick your point and direction vector.)

2. (6 points) Use Gaussian elimination or Gauss-Jordan elimination to solve

$$\begin{cases} x + y - 2z = 4 \\ x + 3y - z = 7 \\ 2x + y - 5z = 7 \end{cases}$$

Solution This solution will use Gaussian elimination. First, write down the augmented matrix of this linear system:

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 1 & 3 & -1 & 7 \\ 2 & 1 & -5 & 7 \end{array}\right]$$

Next, apply elementary row operations to convert this matrix into a row echelon form:

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 1 & 3 & -1 & 7 \\ 2 & 1 & -5 & 7 \end{array}\right] \xrightarrow{row2-row1} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & 2 & 1 & 3 \\ 2 & 1 & -5 & 7 \end{array}\right] \xrightarrow{row3-2row1} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & 2 & 1 & 3 \\ 0 & -1 & -1 & -1 \end{array}\right]$$

$$\xrightarrow{switch\ row2\&3} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & -1 & -1 & -1 \\ 0 & 2 & 1 & 3 \end{array}\right] \xrightarrow{row3+2row2} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & -1 & -1 & -1 \\ 0 & 0 & -1 & 1 \end{array}\right]$$

(Remark: There are more than one way to convert the augmented matrix into a row echelon form.)

The row echelon form tells us that this linear system has a unique solution. Using the backward substitution, we have

$$\begin{cases} x + y - 2z = 4 \\ -y - z = -1 \\ -z = 1 \end{cases} \qquad \Rightarrow \qquad \begin{cases} x = 0 \\ y = 2 \\ z = -1 \end{cases}$$