**Quiz # 5**– Math 2233, Differential Equations – Nov. 13, 2008

1. Compute the inverse Laplace transform

$$\mathcal{L}^{-1}\left\{\frac{3s}{s^2-s-6}\right\}$$

**Solution.** First, by using the partial fractions, we have

$$\begin{aligned}
&\frac{3s}{s^2-s-6} = \frac{3s}{(s+2)(s-3)} = \frac{a}{s+2} + \frac{b}{s-3} \\
\Rightarrow\quad &\frac{3s}{s^2-s-6} = \frac{a(s-3)+b(s+2)}{s^2-s-6} \\
\Rightarrow\quad &3s = a(s-3)+b(s+2) = (a+b)s + (2b-3a) \\
\Rightarrow\quad &a+b=3, \quad 2b-3a=0 \\
\Rightarrow\quad &a = 6/5, \quad b = 9/5
\end{aligned}$$

Therefore, we have

$$\frac{3s}{s^2-s-6} = \frac{6/5}{s+2} + \frac{9/5}{s-3}.$$

Hence

$$\begin{aligned}
\mathcal{L}^{-1}\left\{\frac{3s}{s^2-s-6}\right\} &= \frac{6}{5}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \frac{9}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} \\
&= \frac{6}{5}e^{-2t} + \frac{9}{5}e^{3t}.
\end{aligned}$$

2. Compute the Laplace transform of

$$g(t) = \begin{cases} t & 0 \le t < 1 \\ 1 & t \ge 1 \end{cases}$$

**Solution.** The function can be rewritten as

$$g(t) = t + u_1(t)(1-t).$$

Hence

$$\begin{aligned}
\mathcal{L}\{g(t)\} &= \mathcal{L}\{t + u_1(t)(1-t)\} \\
&= \mathcal{L}\{t\} + \mathcal{L}\{u_1(t)(1-t)\} \\
&\quad (c=1,\ f(t-1) = 1-t \ \Rightarrow\ f(t) = -t) \\
&= \frac{1}{s^2} + e^{-s}\mathcal{L}\{-t\} \\
&= \frac{1}{s^2} - e^{-s}\frac{1}{s^2}
\end{aligned}$$