Quiz # 4– Math 2233, Differential Equations – Oct. 29, 2009

1. (5 points) Find the general solution to

\[ y^{(4)} + 3y'' - 4y = 0 \]

Solution. The characteristic equation is

\[ r^4 + 3r^2 - 4 = 0 \]

\[ \Rightarrow (r^2 + 4)(r^2 - 1) = 0 \]

\[ \Rightarrow (r - 2i)(r + 2i)(r - 1)(r + 1) = 0 \]

\[ \Rightarrow r_1 = 2i, \quad r_2 = -2i, \quad r_3 = 1, \quad r_4 = -1 \]

Hence the general solution is

\[ y = c_1 \cos 2t + c_2 \sin 2t + c_3 e^t + c_4 e^{-t} \]

2. (5 points) Find the Laplace Transform of the solution to the following initial value problem:

\[ \begin{cases} y'' + 3y' + 2y = 0 \\ y(0) = 1, \quad y'(0) = 0 \end{cases} \]

Solution. Apply Laplace transform to both sides of the equation,

\[ L\{y'' + 3y' + 2y\} = L\{0\} = 0 \]

\[ \Rightarrow L\{y''\} + 3L\{y'\} + 2L\{y\} = 0 \]

\[ \Rightarrow (s^2 L\{y\} - sy(0) - y'(0)) + 3(s L\{y\} - y(0)) + 2L\{y\} = 0 \]

Since \( y(0) = 1, \quad y'(0) = 0 \), we have

\[ (s^2 L\{y\} - s) + 3(s L\{y\} - 1) + 2L\{y\} = 0 \]

\[ \Rightarrow (s^2 + 3s + 2)L\{y\} - s - 3 = 0 \]

\[ \Rightarrow L\{y\} = \frac{s + 3}{s^2 + 3s + 2} \]