Quiz # 4– Math 2233, Differential Equations – Oct. 29, 2009

1. (5 points) Find the general solution to

$$y^{(4)} + 3y'' - 4y = 0$$

Solution. The characteristic equation is

$$\begin{aligned} & r^4 + 3r^2 - 4 = 0 \\ \Rightarrow\quad & (r^2+4)(r^2-1) = 0 \\ \Rightarrow\quad & (r-2i)(r+2i)(r-1)(r+1) = 0 \\ \Rightarrow\quad & r_1 = 2i, \quad r_2 = -2i, \quad r_3 = 1, \quad r_4 = -1 \end{aligned}$$

Hence the general solution is

$$y = c_1 \cos 2t + c_2 \sin 2t + c_3 e^t + c_4 e^{-t}$$

2. (5 points) Find the Laplace Transform of the solution to the following initial value problem:

$$\begin{cases} y'' + 3y' + 2y = 0 \\ y(0) = 1, \quad y'(0) = 0 \end{cases}$$

Solution. Apply Laplace transform to both sides of the equation,

$$\begin{aligned} & \mathcal{L}\{y'' + 3y' + 2y\} = \mathcal{L}\{0\} = 0 \\ \Rightarrow\quad & \mathcal{L}\{y''\} + 3\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = 0 \\ \Rightarrow\quad & \left(s^2\mathcal{L}\{y\} - sy(0) - y'(0)\right) + 3\left(s\mathcal{L}\{y\} - y(0)\right) + 2\mathcal{L}\{y\} = 0 \end{aligned}$$

Since $y(0) = 1, \quad y'(0) = 0$, we have

$$\begin{aligned} & \left(s^2\mathcal{L}\{y\} - s\right) + 3\left(s\mathcal{L}\{y\} - 1\right) + 2\mathcal{L}\{y\} = 0 \\ \Rightarrow\quad & (s^2 + 3s + 2)\mathcal{L}\{y\} - s - 3 = 0 \\ \Rightarrow\quad & \mathcal{L}\{y\} = \frac{s+3}{s^2+3s+2} \end{aligned}$$