Quiz # 3– Math 2233, Differential Equations – Oct. 2, 2009

1. Solve the following problem:

$$\begin{cases} y'' - 6y' + 9y = 0 \\ y(0) = 0, \quad y'(0) = 2 \end{cases}$$

Solution First, find the general solution by using the characteristic equation:

$$r^2 - 6r + 9 = 0 \qquad \Rightarrow \qquad (r-3)^2 = 0 \qquad \Rightarrow \qquad r_1 = r_2 = 3$$

We have a repeated root 3, hence the general solution is:

$$y = c_1 e^{3t} + c_2 t e^{3t}$$

By the product rule, we have

$$y' = 3c_1 e^{3t} + c_2 e^{3t} + 3c_2 t e^{3t}$$

Next, use the initial conditions

$$\begin{cases} y(0) = c_1 = 0 \\ y'(0) = 3c_1 + c_2 = 2 \end{cases} \qquad \Rightarrow \qquad \begin{cases} c_1 = 0 \\ c_2 = 2 \end{cases}$$

Therefore, the particular solution is

$$y = 2te^{3t}$$

2. Find the general solution to

$$y'' - y' - 2y = 4t$$

Solution First, we need to solve the associated homogeneous equation

$$y'' - y' - 2y = 0$$

It has characteristic equation $r^2 - r - 2 = (r+1)(r-2) = 0$. Therefore $r_1 = -1$ and $r_2 = 2$ are two distinct real roots, and the general solution to the homogeneous equation is

$$y = c_1 e^{-t} + c_2 e^{2t}$$

Next, we need to find a specific solution Y to the nonhomogeneous equation. By the method of undetermined coefficients, a proper form for Y is $Y = At+B$. Since 0 is not a root to the characteristic equation, we do not need to multiply Y by an extra t.

Substitute

$$Y = At + B, \qquad Y' = A, \qquad Y'' = 0$$

into the nonhomogeneous equation gives

$$\begin{aligned}
& 0 - A - 2(At + B) = 4t \\
\Rightarrow \qquad & -A - 2At - 2B = 4t \\
\Rightarrow \qquad & -2At + (-A - 2B) = 4t + 0 \\
\Rightarrow \qquad & \begin{cases} -2A = 4 \\ -A - 2B = 0 \end{cases} \qquad \Rightarrow \qquad \begin{cases} A = -2 \\ B = 1 \end{cases}
\end{aligned}$$

Therefore the specific solution $Y = -2t + 1$.

Combining the above, the general solution to the nonhomogeneous equation is

$$y = c_1 e^{-t} + c_2 e^{2t} - 2t + 1$$