Quiz # 1– Math 2233, Differential Equations – Aug 28, 2009

1. (4 pts) Determine the values of r such that $y = t^r$, for $t > 0$, is a solution to

$$t^2 y'' + 4ty' + 2y = 0.$$

Solution Substitute $y = t^r$ into the equation, we have

$$\begin{aligned} & t^2\left(r(r-1)t^{r-2}\right) + 4t\left(rt^{r-1}\right) + 2t^r = 0 \\ \Rightarrow\ & r(r-1)t^r + 4rt^r + 2t^r = 0 \\ \Rightarrow\ & (r^2+3r+2)t^r = 0 \\ \Rightarrow\ & r^2+3r+2 = 0 \\ \Rightarrow\ & r = -1 \text{ or } r = -2 \end{aligned}$$

2. (6 pts) Solve the following differential equation when $t > 0$:

$$\begin{cases} ty' + (t+1)y = 1, \\ y(1) = 1 + e^{-1}. \end{cases}$$

Solution

$$\begin{aligned} & y' + \frac{t+1}{t}y = \frac{1}{t} \\ & \mu(t) = e^{\int \frac{t+1}{t}\,dt} = e^{t+\ln t} = te^t \qquad \text{since } t > 0 \\ & (te^t y)' = te^t\frac{1}{t} = e^t \qquad \text{multiply by } \mu(t) \text{ on both sides} \\ & te^t y = e^t + c \qquad \text{integrate both sides} \\ & y = \frac{1}{t} + \frac{c}{te^t} \end{aligned}$$

This is the general solution. To get the particular solution, use $y(1) = 1 + e^{-1}$. Then

$$\begin{aligned} 1 + e^{-1} &= \frac{1}{1} + \frac{c}{1 \cdot e^1} \\ &\Rightarrow\ c = 1 \end{aligned}$$

Therefore the particular solution is

$$y = \frac{1}{t} + \frac{1}{te^t}$$