

Quiz # 9– Math 2233, Differential Equations – Nov. 6, 2008

1. Use $\mathcal{L}\{u_c(t)f(t-c)\} = e^{-cs}F(s)$ to find the Laplace transform of

$$g(t) = \begin{cases} t & 0 \leq t < 1 \\ 1 & t \geq 1 \end{cases}$$

Formula: $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$ for $s > 0$.

Solution. The function can be rewritten as

$$g(t) = t + u_1(t)(1-t).$$

Hence

$$\begin{aligned} \mathcal{L}\{g(t)\} &= \mathcal{L}\{t + u_1(t)(1-t)\} \\ &= \mathcal{L}\{t\} + \mathcal{L}\{u_1(t)(1-t)\} \\ &\quad (c=1, f(t-1) = 1-t \Rightarrow f(t) = -t) \\ &= \frac{1}{s^2} + e^{-s}\mathcal{L}\{-t\} \\ &= \frac{1}{s^2} - e^{-s}\frac{1}{s^2} \end{aligned}$$