

**Quiz # 8**– Math 2233, Differential Equations – Oct. 30, 2008

1. Given the Laplace transform

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

for  $s > a$ . Find the inverse Laplace transform

$$\mathcal{L}^{-1}\left\{\frac{3s}{s^2 - s - 6}\right\}$$

**Solution.** First, by using the partial fractions, we have

$$\begin{aligned}\frac{3s}{s^2 - s - 6} &= \frac{3s}{(s+2)(s-3)} = \frac{a}{s+2} + \frac{b}{s-3} \\ \Rightarrow \frac{3s}{s^2 - s - 6} &= \frac{a(s-3) + b(s+2)}{s^2 - s - 6} \\ \Rightarrow 3s &= a(s-3) + b(s+2) = (a+b)s + (2b-3a) \\ \Rightarrow a+b &= 3, \quad 2b-3a = 0 \\ \Rightarrow a &= 6/5, \quad b = 9/5\end{aligned}$$

Therefore, we have

$$\frac{3s}{s^2 - s - 6} = \frac{6/5}{s+2} + \frac{9/5}{s-3}.$$

Hence

$$\begin{aligned}\mathcal{L}^{-1}\left\{\frac{3s}{s^2 - s - 6}\right\} &= \frac{6}{5}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \frac{9}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} \\ &= \frac{6}{5}e^{-2t} + \frac{9}{5}e^{3t}.\end{aligned}$$