Quiz # 5– Math 2233, Differential Equations – Oct. 2, 2008

1. Find the general solution to

$$y'' - y' - 2y = 4t + \sin t - 3\cos t$$

Solution: We solve the problem in following steps:

Step 1. Compute y_1, y_2 which are solutions to the associated homogeneous equation $y'' - y' - 2y = 0$. Notice that the characteristic equation is

$$r^2 - r - 2 = 0 \quad \Rightarrow (r-2)(r+1) = 0$$

We have $r_1 = 2$ and $r_2 = -1$. Hence

$$y_1 = e^{2t}, \qquad y_2 = e^{-t}$$

Step 2. Compute a specific solution Y by using the method of undetermined coefficients. A proper form of Y is

$$Y = (At + B) + (C\cos t + D\sin t)$$

Then

$$\begin{aligned} Y' &= A + (-C\sin t + D\cos t) \\ Y'' &= -C\cos t - D\sin t \end{aligned}$$

Substitute them in to the non-homogeneous equation, we have

$$\begin{aligned} &[-C\cos t - D\sin t] - [A + (-C\sin t + D\cos t)] \\ &\qquad - 2[(At + B) + (C\cos t + D\sin t)] = 4t + \sin t - 3\cos t \\ \Rightarrow \quad &(-2At - A - 2B) + (-3D + C)\sin t + (-3C - D)\cos t \\ &\qquad = 4t + \sin t - 3\cos t \\ \Rightarrow \quad &\begin{cases} -2A = 4 \\ -A - 2B = 0 \end{cases} \qquad \begin{cases} -3D + C = 1 \\ -3C - D = -3 \end{cases} \\ \Rightarrow \quad &A = -2 \quad B = 1 \quad C = 1 \quad D = 0 \end{aligned}$$

Therefore, the specfic solution is

$$Y = -2t + 1 + \cos t$$

Step 3. Combining the above, the general solution is

$$y = c_1 y_1 + c_2 y_2 + Y = c_1 e^{2t} + c_2 e^{-t} - 2t + 1 + \cos t$$