**Quiz # 2**– Math 2233, Differential Equations – Sep. 4, 2008

1. A tank initially contains 200 gallons of salt water with a concentration of 2 oz/gal. Pure water flows into the tank at a rate of 10 gal/min, while the well-stirred mixture flows out of the tank at a rate of 5 gal/min. Find the total amount of salt in the tank after 10 minutes.

Solution: Let $Q(t)$ be the amount of salt in the tank at time $t$. Then we have

$$\begin{aligned}\frac{dQ}{dt} &= \text{rate of salt in} - \text{rate of salt out} \\ &= 10 \times 0 - 5 \times \frac{Q}{200+5t} = -\frac{5}{200+5t}Q = -\frac{1}{40+t}Q\end{aligned}$$

And at time $t = 0$, we have the initial condition:

$$Q(0) = 200\,(gal) \times 2\,(oz/gal) = 400\,(oz)$$

Combining them together gives the following initial value problem (IVP):

$$\begin{cases} \frac{dQ}{dt} = -\frac{1}{40+t}Q \\ Q(0) = 400 \end{cases}$$

To solve the IVP, we first notice that the differential equation is separable,

$$\begin{aligned}\frac{1}{Q}dQ = -\frac{1}{40+t}dt \quad &\Rightarrow \quad \ln|Q| = -\ln|40+t| + c \\ &\Rightarrow \quad \ln|Q| = \ln\frac{C}{|40+t|} \quad \text{where } C = e^c \\ &\Rightarrow \quad Q = \frac{C}{40+t}\end{aligned}$$

Then using the initial condition,

$$400 = \frac{C}{40+0} \quad \Rightarrow \quad C = 16000$$

So the particular solution is $Q(t) = \frac{16000}{40+t}$ and at $t = 10$mins,

$$Q(10) = \frac{16000}{40+10} = 320$$