Quiz # 11– Math 2233, Differential Equations – Due: Dec. 4, 2008

1. Determine the radius of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{n^2(x+2)^n}{3^n}$$

Solution By using the root test, we have

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^2(x+2)^{n+1}}{3^{n+1}}}{\frac{n^2(x+2)^n}{3^n}} \right| = \lim_{n\to\infty} \left| \frac{(x+2)(n+1)^2}{3n^2} \right| = \left| \frac{x+2}{3} \right| \lim_{n\to\infty} \left| \frac{(n+1)^2}{n^2} \right|$$

$$= \left| \frac{x+2}{3} \right| \lim_{n\to\infty} \left| (1+\frac{1}{n})^2 \right| = \left| \frac{x+2}{3} \right|$$

The series is convergent when

$$\left| \frac{x+2}{3} \right| < 1 \quad \Rightarrow \quad |x+2| < 3.$$

Therefore the radius of convergence is 3.