1. Determine the order of following differential equations. Also state whether they are linear or nonlinear.

   (a) $yy' + t = 1$
   (b) $ty' + y = 1$
   (c) $(y')^3 + ty = 1$
   (d) $y''' + \sqrt{t}y = 1$

2. Show that $y = t^3$ is a solution of the initial value problem $y' = 3y^{2/3}$, $y(0) = 0$. (Can you find a different solution for this problem?)

3. Solve the following differential equations:

   (a) $y' = e^{2y}t$
   (b) $y' = \frac{x^4+y^4}{xy^3}$
   (c) $(x + 2y) + (2x + y)y' = 0$ (homogeneous, need to use “completion of squares” to integrate)
   (d) $(x + 2y) - (2x + y)y' = 0$ (homogeneous, need to use “partial fractions” to integrate)
   (e) $(2xy + 1)dy + (x^2 + y^2)dx = 0$ (section 2.6)
   (f) $(ye^{xy}\cos(2x) - 2e^{xy}\sin(2x) + 2x)dx + (xe^{xy}\cos(2x) - 3)dy = 0$ (section 2.6)
   (g) $xy' + 3y = 5x^2$
   (h) $y' = \frac{y}{x} + e^{y/x}$ (substitution $v = y/x$)
   (i) $y' = (3x^2 - e^x)/(2y - 5)$
   (j) $\sin(2x)dx + \cos(3y)dy = 0$
   (k) $y'' + y' = e^{-t}$

4. Determine the interval in which the solution is valid. (For linear equations, use Theorem 2.4.1 on page 68. For nonlinear equations, you will need to check both the differential equation and the solution.)

   (a) $(4 - t^2)y' + 2ty = 3t^2$, $y(1) = -3$
   (b) $\frac{(t+1)(t-3)}{t-2}y' + \frac{t-3}{t(t-2)}y = \frac{(t+1)(t+3)}{t}$, $y(1) = 0$
   (c) $(\sin t)y' + \frac{t}{t-1}y = \ln(t - 2)$, $y(3) = 0$
   (d) $y' = \frac{3x^2+4x+2}{2(y-1)}$, $y(0) = -1$
   (e) $\frac{dr}{d\theta} = r^2/\theta$, $r(1) = 2$

5. (section 2.5) Determine the critical (equilibrium) points, and classify each one as asymptotically stable, unstable or semistable. Draw the phase line and sketch several graphs of solutions.

   (a) $y' = -(y - 1)^2$

(b) $y' = y(1-y^2)$

(c) $y' = y^2(4-y^2)$

(d) $y' = (y-1)^2(y+2)(y-4)$

6. Modelling problems (try to solve as many problems as you can from section 2.3 exercises, page 59-68).

Answer (Please let me know if you find any mistake)

1. (a) 1st order nonlinear; (b) 1st, linear; (c) 1st, nonlinear; (d) 3rd, linear
2. Another solution is $y = 0$
3. (a) $e^{-2y}+t^2 = C$; (b) $(y/x)^4 = 4\ln|x|+C$; (c) $x^2+4xy+y^2 = C$; (d) $\ln|1+y/x|-3\ln|1-y/x| = 2\ln|x|+C$; (e)$xy^2+y+x^3/3 = C$ (f) $x^{xy}\cos(2x)+x^2-3y = C$; (g) $x^3y = x^5+C$; (h) $e^{-y/x}+\ln x = C$; (i) $y^2-5y = x^3-e^x+C$; (j) $-\cos(2x)/2+\sin(3y)/3) = C$; (k) $y = c_1e^{-t}+c_2-te^{-t}$.
4. (a) $-2 < t < 2$; (b) $0 < t < 2$; (c) $2 < t < \pi$; (d) Page 44, example 2; (e) $0 < \theta < \sqrt{e}$.
5. (a) y=1 is semistable; (b) y=0 is unstable, y=1 is stable, y=-1 is stable; (c) y=-2 is unstable, y=0 is semistable, y=2 is stable; (d) y=-2 is stable, y=1 is semistable, y=4 is unstable.