Math 2163, Practice Exam III

1. Sketch the 2-D regions:

   (a) $D = \{(x, y)\,|\,0 \le y \le 2,\ 0 \le x \le 2 - y\}$,

   (b) $D = \{(x, y)\,|\,-2 \le x \le 2,\ -x^2 - 1 \le y \le \sqrt{4 - x^2}\}$

   (c) $D = \{(r, \theta)\,|\,0 \le \theta \le 2\pi,\ 1 \le r \le 6\}$,

   (d) $D = \{(r, \theta)\,|\,-\pi/4 \le \theta \le \pi/4,\ 0 \le r \le \cos 2\theta\}$.

2. Sketch the 3-D regions:

   (a) $D = \{(x, y, z)\,|\,-1 \le y \le 1,\ y^2 \le x \le 1,\ 0 \le z \le x\}$,

   (b) $D = \{(x, y, z)\,|\,1 \le x \le 2,\ 0 \le y \le \sqrt{4 - x^2},\ 0 \le z \le \sqrt{1 - x^2/4 - y^2/4}\}$,

   (c) $D = \{(r, \theta, z)\,|\,0 \le \theta \le \pi/2,\ 0 \le r \le 1,\ 1 - r^2 \le z \le 1\}$,

   (d) $D = \{(\rho, \theta, \phi)\,|\,1 \le \rho \le 2,\ \pi/2 \le \theta \le \pi, 0 \le \phi \le \pi\}$.

3. Evaluate iterated integrals:

   (a) $\int_0^1 \int_0^1 y e^{xy}\, dx\, dy$,

   (b) $\int_0^1 \int_0^y \int_x^1 6xyz\, dz\, dx\, dy$,

   (c) $\int_0^{\pi/2} \int_0^{\sin 2\theta} r\, dr\, d\theta$,

   (d) $\int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$.

4. Find the area of the region enclosed by the curve $r = 4 + 3\cos\theta$.

5. Find the area of the part of the surface $z = x^2 + y^2$ lies inside the circle $x^2 + y^2 = 9$.

6. Evaluate the volume bounded by paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

7. Find the mass and center of the mass of the solid tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 3)$, with density function $\rho(x, y, z) = x^2 + y^2 + z^2$.

8. Evaluate the triple integrals

   (a) $\iiint_E xy\, dV$, where $E = \{(x, y, z)\,|\,0 \le x \le 3,\ 0 \le y \le x,\ 0 \le z \le x + y\}$,

   (b) $\iiint_E yz\, dV$, where E is bounded by $z = 0$, $z = y$ and lies inside the cylinder $x^2 + y^2 = 4$, $y \ge 0$,

   (c) $\iiint_E z^3\sqrt{x^2 + y^2 + z^2}\, dV$ where E is the hemisphere that lies above the xy-plane and has center the origin and radius 1.

9. Find the Jacobian of the transformation

$$x = e^{s+t}, \qquad y = e^{s-t}$$

10. Use the given transformation to evaluate the integral

$$\iint_R x^2 \, dA$$

where R is the region bounded by the ellipse $9x^2 + 4y^2 = 36$, use the transformation $x = 2u$, $y = 3v$.

11. Find and sketch the gradient vector field of

$$f(x, y) = xe^{xy}$$

12. Find the gradient vector field of

$$f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$$