Math 2163, Practice Exam II

1. Find the directional derivative of the functions at the given point in the given direction:

   - $f(x, y) = s^2 e^t$, $(2, 0)$, $\mathbf{v} = \mathbf{i} + \mathbf{j}$,
   - $f(x, y) = x^2y^3 - y^4$, $(2, 1)$, $\theta = \pi/4$,
   - $f(x, y, z) = x/(y + z)$, $(4, 1, 1)$, $\mathbf{v} =< 1, 2, 3 >$.

2. Find the maximum rate of change of f at the given point:

   - $f(x, y) = y^2/x$, $(2, 4)$,
   - $f(x, y, z) = x^4y^3z^2$, $(1, -1, 1)$.

3. Find and classify all critical points of

   - $f(x, y) = (x^2 + y^2)e^{y^2 - x^2}$,
   - $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$.

4. Use Lagrange multipliers to find the shortest distance from the point $(8, 10, 8)$ to the plane $8x - 10y + 4z = 16$.

5. Use Lagrange multipliers to find the dimension of the rectangular box with largest volume if the total surface area is given as $150\ cm^2$.

6. Find an approximation for the integral

$$\iint_R (4x - 5y^2)\ dA, \qquad R = [0, 8] \times [04]$$

   by a double Riemann sum with $m = n = 2$ and the sample point in upper right corner.

7. Calculate the double integrals:

   - $\int_0^6 \int_0^{10} \sqrt{(x + y)}\ dx\, dy$,
   - $\iint_D \frac{3y}{x^2+1} dA$, $D = \{0 \le x \le 9,\ 0 \le y \le \sqrt{x}\}$,
   - $\int_1^2 \int_0^1 \frac{x}{x^2+y^2}\ dy\, dx$,
   - $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1}\, dx\, dy$.

8. Sketch the 2-D regions, and evaluate the double integral on these regions.

   (a) $D = \{(x, y) \mid 0 \le y \le 2,\ 0 \le x \le 2 - y\}$, find $\iint_D y \cos x\, dA$

   (b) $D = \{(x, y) \mid -2 \le x \le 2,\ -x^2 - 1 \le y \le \sqrt{4 - x^2}\}$, find $\iint_D (x + 2y)\, dA$