Math 2163, Practice Exam I

1. Find $|\mathbf{a}|$, $3\mathbf{a}+4\mathbf{b}$, $\mathbf{a}\cdot\mathbf{b}$, $\mathbf{a}\times\mathbf{b}$ where

   (a) $\mathbf{a} =< -3, 1, 2 >$, $\mathbf{b} =< 6, 1, 7 >$;

   (b) $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{j} + 2\mathbf{k}$.

2. Find a unite vector that has the same direction as $12\mathbf{i} - 5\mathbf{j}$.

3. Find the angle between vectors $\mathbf{a}$ and $\mathbf{b}$, determine whether they are orthogonal, parallel, or skew:

   (a) $\mathbf{a} =< 6, -3, 2 >$, $\mathbf{b} =< 2, 1, -2 >$;

   (b) $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$;

   (c) $\mathbf{a} =< 4, 6 >$, $\mathbf{b} =< -3, 2 >$;

   (d) $\mathbf{a} = 2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$, $\mathbf{b} = -3\mathbf{i} - 9\mathbf{j} + 6\mathbf{k}$;

   (e) $\mathbf{a} =< a, b, c >$, $\mathbf{b} =< -b, a, 0 >$.

4. Find the direction cosines of the vector $< 1, -2, -1 >$.

5. Find the scalar and vector projection of $< -1, -2, 2 >$ onto $< 3, 3, 4 >$.

6. Find the two unit vectors orthogonal to both $\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $2\mathbf{i} + \mathbf{k}$.

7. Find the volume of the parallelpiped decided by $< 1, 1, -1 >$, $< 1, -1, 1 >$ and $< -1, 1, 1 >$.

8. Find the lines specified by:

   (a) passes through the point $(1, 0, -3)$ and parallel to the vector $< 2, -4, 5 >$;

   (b) passes through the point $(-2, 4, 1)$ and perpendicular to the plane $x+y-z = 5$;

   (c) passes through two points $(1, 1, 3)$ and $(2, -1, 0)$;

   (d) passes through the point $(2, 1, 0)$ and perpendicular to both $\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{i} - \mathbf{j}$;

   (e) the line of intersection of the planes $x + y + z = 1$ and $x + z = 0$.

9. Find the plane specified by:

   (a) passes through the point $(4, 1, 2)$ and perpendicular to the vector $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$;

   (b) passes through the point $(4, 1, 2)$ and parallel to the plane $x - y + z = 5$;

   (c) pases through three points $(3, -1, 2)$, $(8, 2, 4)$ and $(-1, -2, -3)$;

   (d) passes through a point $(1, 3, 0)$ and contains the line $x = 1 + t$, $y = 2t$, $z = -1 - t$;

10. Find the unit tangent, unit normal, and unit binormal vectors for the curve. And compute the curvature.

(a) $\mathbf{r}(t) =< 2\sin t,\ 5t,\ 2\cos t >$

(b) $\mathbf{r}(t) =< \sqrt{t},\ e^t,\ e^{-t} >$

11. Find the length of the curve

(a) $\mathbf{r}(t) =< 2\sin t,\ 5t,\ 2\cos t >$, $-10 \le t \le 10$.

(b) $\mathbf{r}(t) =< 2t,\ t^2,\ t^3/3 >$, $0 \le t \le 1$.

12. Find the domain of the following functions:

(a) $f(x, y) = \sqrt{y - x}\ln(y + x)$;

(b) $f(x, y) = \sqrt{16 - x^2 - 16y^2}$;

(c) $f(x, y) = \frac{x-3y}{x^2-y}$

13. Show that the following limits does not exist:

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + 2y^2}{2x^2 + y^2}$;

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^4 + y^4}$;

(c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{sin(xy)}{x^2 - y^2}$

14. Find all first order partial derivatives for:

(a) $f(x, y) = x^y$;

(b) $f(x, y) = \frac{xy^2}{x^2+y^2}$;

(c) $f(x, y) = \int_x^y \cos t\,dt$;

(d) $f(x, y, z) = x^2e^{yz}$;

(e) $f(x, y, z) = x\tan(xy)$;

(f) $z = \sin u$ and $u = y/x$;

(g) $z = xy + y^2$ and $x = 2t$, $y = 1 - t$;

(h) $z = xy + yz + zx$ and $x = st$, $y = e^{st}$, $z = t^2$;

(i) $xy^2 + yz^2 + zx^2 = 3$ and z is a function of x and y;

(j) $xyz = \cos(xyz)$ and z is a function of x and y;

15. Find the following higher order derivatives:

(a) $z = x\sin y$, find $\frac{\partial^2 z}{\partial y^2}$;

(b) $f(x, y) = e^{xy^2}$, find f_{yy};

(c) $f(r, s, t) = r\ln(rs^2t^3)$, find f_{rss} and f_{rst}.

16. Find the total differential dz of $z = e^s \sin t$.

17. Given $z = \ln(2x + y)$, find the tangent plane at $(-1, 3, 0)$.

18. Find the linear approximation of $f(x, y) = xe^{xy}$ at $(1, 0)$ and use it to approximate $f(1.1, -0.1)$.

19. Use the chain rule to find the partive derivatives:

    (a) $u = \sqrt{r^2 + s^2}$, $r = y + x \cos t$, $s = x + y \sin t$, find $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial u}{\partial t}$, when $x = 1$, $y = 2$, $t = 0$.

    (b) $M = xe^{y-z^2}$, $x = 2uv$, $y = u - v$, $z = u + v$, find $\frac{\partial M}{\partial u}$, $\frac{\partial M}{\partial v}$, when $u = 3$, $v = 1$.