Quiz # 9– Math 2163, Calculus III – Nov. 2, 2007

Show all your work neatly and concisely, and indicate your final answer clearly.

1. Use polar coordinates to find the volume bounded by the paraboloid $z = 2x^2 + 2y^2$ and the plane $z = 2$.

   Solution: The volume is

$$\iint_D [(\text{top surface}) - (\text{bottom surface})]\,dA = \iint_D [2 - (2x^2 + 2y^2)]\,dA$$

   where D is the disk defined by the intersection of the paraboloid and the plane:

$$\begin{cases} z = 2x^2 + 2y^2 \\ z = 2 \end{cases} \qquad \Rightarrow \qquad x^2 + y^2 = 1.$$

   In polar coordinates, the disk inside $x^2 + y^2 = 1$ is

$$D = \{(r, \theta) \,|\, 0 \le \theta \le 2\pi,\ 0 \le r \le 1\},$$

   and the integrand $2 - (2x^2 + 2y^2)$ can be written as

$$2 - (2x^2 + 2y^2) = 2 - 2(x^2 + y^2) = 2 - 2r^2.$$

   Putting these together, we get the volume

$$\begin{aligned} \iint_D [2 - (2x^2 + 2y^2)]\,dA &= \int_0^{2\pi} \int_0^1 [2 - 2r^2] r\,dr\,d\theta \\ &= \int_0^{2\pi} [r^2 - \frac{1}{2} r^4]|_{r=0}^{r=1}\,d\theta \\ &= \int_0^{2\pi} \frac{1}{2}\,d\theta \\ &= \frac{1}{2}\theta|_{\theta=0}^{\theta=2\pi} = \pi. \end{aligned}$$