

Quiz # 7– Math 2163, Calculus III – Oct. 12, 2007

Show all your work neatly and concisely, and indicate your final answer clearly.

1. Calculate the double integral

$$\iint_R e^{2x-y} dA, \quad R = [0, \ln 2] \times [0, \ln 5].$$

Solution 1:

$$\begin{aligned} \iint_R e^{2x-y} dA &= \iint_R e^{2x} e^{-y} dA = \left(\int_0^{\ln 2} e^{2x} dx \right) \left(\int_0^{\ln 5} e^{-y} dy \right) \\ &= \left(\frac{1}{2} e^{2x} \Big|_0^{\ln 2} \right) \left(-e^{-y} \Big|_0^{\ln 5} \right) = (e^{2 \ln 2} / 2 - 1/2) (-e^{-\ln 5} - (-1)) = (2 - 1/2) (-1/5 + 1) = 6/5 \end{aligned}$$

Solution 2:

$$\begin{aligned} \iint_R e^{2x-y} dA &= \int_0^{\ln 2} \int_0^{\ln 5} e^{2x-y} dy dx \\ &= \int_0^{\ln 2} \left(-e^{2x-y} \Big|_{y=0}^{y=\ln 5} \right) dx = \int_0^{\ln 2} (e^{2x} - e^{2x-\ln 5}) dx \\ &= (e^{2x} / 2 - e^{2x-\ln 5} / 2) \Big|_0^{\ln 2} \\ &= (e^{2 \ln 2} / 2 - e^{2 \ln 2 - \ln 5} / 2) - (e^0 / 2 - e^{-\ln 5} / 2) \\ &= (2 - 2/5) - (1/2 - 1/10) = 6/5 \end{aligned}$$

Solution 3:

$$\begin{aligned} \iint_R e^{2x-y} dA &= \int_0^{\ln 5} \int_0^{\ln 2} e^{2x-y} dx dy \\ &= \int_0^{\ln 5} \left(\frac{1}{2} e^{2x-y} \Big|_{x=0}^{x=\ln 2} \right) dy \\ &= \int_0^{\ln 5} (e^{2 \ln 2 - y} / 2 - e^{-y} / 2) dy \\ &= (-e^{2 \ln 2 - y} / 2 + e^{-y} / 2) \Big|_0^{\ln 5} \\ &= (-e^{2 \ln 2 - \ln 5} / 2 + e^{-\ln 5} / 2 - (-e^{2 \ln 2} / 2 + e^0 / 2)) \\ &= (-2/5 + 1/10) - (-2 + 1/2) = 6/5 \end{aligned}$$