Quiz # 6– Math 2163, Calculus III – Oct. 5, 2007

Show all your work neatly and concisely, and indicate your final answer clearly.

1. Use Lagrange multipliers to find the maximum and the minimum values of the function $f(x,y) = 4x + 6y$ subject to the given constraint $x^2 + y^2 = 13$.

   Solution: we need to find the maximum/minimum values of $f(x,y)$ subject to the constraint $g(x,y) = x^2 + y^2 = 13$. Notice that

$$\nabla f =< 4, 6 >,$$
$$\nabla g =< 2x, 2y > .$$

   Using the Lagrange multiplier,

$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y) = k \end{cases} \quad \Rightarrow \quad \begin{cases} 4 = 2\lambda x \\ 6 = 2\lambda y \\ x^2 + y^2 = 13 \end{cases}$$

   To solve the above system of three equations and three unknowns, we observe from the 1st and the 2nd equation that

$$x = \frac{2}{\lambda}, \qquad y = \frac{3}{\lambda}$$

   Substitute them into the 3rd equation gives

$$(\frac{2}{\lambda})^2 + (\frac{3}{\lambda})^2 = 13, \qquad \Rightarrow \quad \lambda = \pm 1.$$

   - If $\lambda = 1$, then we easily get

$$x = 2, \quad y = 3, \quad f(2,3) = 26$$

   - If $\lambda = -1$, then we easily get

$$x = -2, \quad y = -3, \quad f(-2,-3) = -26$$

   So the final answer is:

   the maximum value occurs at $f(2,3) = 26$
   the manimum value occurs at $f(-2,-3) = -26$