

Quiz # 5– Math 2163, Calculus III – Sept. 28, 2007

Show all your work neatly and concisely, and indicate your final answer clearly.

1. Find the maximum rate of change of $f(x, y, z) = x^2y^3z^4$ at the point $(1, 1, 1)$. In what direction does f have the maximum rate of change?

Solution: First calculate the gradient vector

$$\nabla f = \langle f_x, f_y, f_z \rangle = \langle 2xy^3z^4, 3x^2y^2z^4, 4x^2y^3z^3 \rangle .$$

Then

$$\nabla f(1, 1, 1) = \langle 2, 3, 4 \rangle .$$

Therefore, the maximum rate of change is

$$| \langle 2, 3, 4 \rangle | = \sqrt{29}$$

and it occurs in the same direction as $\nabla f(1, 1, 1) = \langle 2, 3, 4 \rangle$, or you can denote the direction by the unit vector

$$\left\langle \frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}} \right\rangle$$