

Quiz # 3– Math 2163, Calculus III – Sept. 7, 2007

Show all your work neatly and concisely, and indicate your final answer clearly.

1. Given

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2 + y^8},$$

try to calculate its value along different paths :

- (a) (1 pt) Approach $(0, 0)$ along the y-axis;
- (b) (1 pt) Approach $(0, 0)$ along $y = x$;
- (c) (1 pt) Approach $(0, 0)$ along $x = y^2$;
- (d) (1 pt) Approach $(0, 0)$ along $x = y^4$;

According to the above calculation, does the limit exist? (1 pt)

Solution:

$$(a) \lim_{y=0, x \rightarrow 0} \frac{x0^4}{x^2 + 0^8} = \lim_{y=0, x \rightarrow 0} 0 = 0;$$

$$(b) \lim_{y=x, (x,y) \rightarrow (0,0)} \frac{x^5}{x^2 + x^8} = \lim_{y=x, (x,y) \rightarrow (0,0)} \frac{x^3}{1 + x^6} = \frac{0^3}{1 + 0^6} = 0;$$

$$(c) \lim_{x=y^2, (x,y) \rightarrow (0,0)} \frac{y^6}{y^4 + y^8} = \lim_{x=y^2, (x,y) \rightarrow (0,0)} \frac{y^2}{1 + y^4} = \frac{0^2}{1 + 0^4} = 0;$$

$$(d) \lim_{x=y^4, (x,y) \rightarrow (0,0)} \frac{y^8}{y^8 + y^8} = \lim_{x=y^4, (x,y) \rightarrow (0,0)} \frac{1}{2} = \frac{1}{2};$$

According to the above calculation, the limit does not exist since we have different values along different paths.